P
Univ.
I

FIRST SERIES, NO. 8 JULY, 1917

UNIVERSITY OF IOWA MONOGRAPHS

CONTRIBUTIONS FROM THE PHYSICAL LABORATORY

VOLUME II NUMBER 1

COLLECTED STUDIES AND REPORTS

PUBLISHED BY THE UNIVERSITY, IOWA CITY, IOWA

ISSUED MONTHLY THROUGHOUT THE YEAR. ENTERED AT THE POSTOFFICE AT IOOWA CITY, IOWA, AS SECOND CLASS MATTER

UNIVERSITY OF IOWA MONOGRAPHS

CONTRIBUTIONS
FROM THE
PHYSICAL LABORATORY

PROFESSOR GEORGE WALTER STEWART, EDITOR

VOLUME II NUMBER 1

CONTENTS

1. *An extension towards the ultra-violet of the wave-length-sensibility curves for certain crystals of metallic selenium*
L. P. SIEG and F. C. BROWN
Reprinted from the PHYSICAL REVIEW, N. S., V. (1915), No. 1

2. *Notes on electrode and diffusion potentials*
G. W. MOFFITT
Reprinted from the PHYSICAL REVIEW, N. S., V. (1915), No. 1

3. *The electrical, the photo-electrical and the electro-mechanical properties of certain crystals of metallic selenium, with certain applications to crystal structure.*
F. C. BROWN
Reprinted from the PHYSICAL REVIEW, N. S., V. (1915), No. 2

4. *Isolated crystals of selenium of the second and fifth systems, and the physical conditions determining their production.*
F. C. BROWN
Reprinted from the PHYSICAL REVIEW, N. S., V. (1915), No. 3

5. *The change in elasticity of a mild steel wire with current and external heating.* H. L. DODGE
Reprinted from the PHYSICAL REVIEW, N. S., V. (1915), No. 5

6. *The nature of electric conduction as required to explain the recovery of resistance of metallic selenium following illumination.* F. C. Brown

Reprinted from the Physical Review, N. S., V (1915), No. 5

7. *Some experiments on the nature of transmitted light-action in crystals of metallic selenium.* F. C. Brown

Reprinted from the Physical Review, N. S., V (1915), No. 5

8. *An attempt to detect a change in the heat conductivity of a selenium crystal with a change in illumination.* L. P. Sieg

Reprinted from the Physical Review, N. S., VI (1915), No. 3

9. *The change in the elasticity of aluminum wire with current and external heating.* H. L. Dodge

Reprinted from the Physical Review, N. S., VI (1915), No. 4

10. *Certain cases of the variation of sound intensity with distance.* G. W. Stewart

Reprinted from the Physical Review, N. S., VII (1916), No. 4

11. *The effect of temperature on the resistance, the light-sensitiveness, and the rate of recovery of certain crystals of metallic selenium.* Kathryn Johnstone Dieterich

Reprinted from the Physical Review, N. S., VII (1916), No. 5

12. *The effect of temperature on the light-sensibility curves of different types of selenium cells.* E. O. Dieterich

Reprinted from the Physical Review, N. S., VIII (1916), No. 2

13. *Notes on the elastic properties of phosphor-bronze wires.* Arnold J. Oehler

Reprinted from the Physical Review, N. S., VIII (1916), No. 5

14. *The frequent bursting of hot water pipes in household plumbing systems.* F. C. Brown

Reprinted from the Physical Review, N. S., VIII (1916), No. 5

15. *Table of* Log_{10} Sec^2 θ *for determining photographic densities by means of Nicol prisms.* Paul S. Helmick

Reprinted from University of Iowa Monographs, No. 6, Series on aims and progress of research.

[Reprinted from the PHYSICAL REVIEW, N.S., Vol. V, No. 1, January, 1915.]

AN EXTENSION TOWARD THE ULTRA-VIOLET OF THE WAVE-LENGTH-SENSIBILITY CURVES FOR CERTAIN CRYSTALS OF METALLIC SELENIUM.

BY L. P. SIEG AND F. C. BROWN.

IN a recent paper[1] we published a number of characteristic wave-length-sensibility curves for certain crystals of metallic selenium. We were unable with the apparatus then employed to carry our curves below wave-length .50 μ with any great degree of certainty, on account of the small amount of energy available from the Nernst glower in this region. We have recently employed a right angled arc, focusing the positive crater of the arc upon the slit. By means of this intrinsically brighter source we have been able to go to the limit of Hilger's monochromatic illuminator, .38 μ. A new instrument that will enable us to work in the ultra-violet region has been ordered, and as soon as it is received we propose to explore these crystals still further into this region. On account of the delay that may be involved at this time in getting this instrument we have thought it best to publish the work as far as we have carried it.

The apparatus employed was the same as that described in our previous paper[2] with the two exceptions of the arc light mentioned above, and a new Leeds and Northrup galvanometer substituted for the Siemens and Halske Panzer galvanometer. The arc light was much less steady than the Nernst glower, but by careful adjustment and attention we were able to get results that were sufficiently reliable to represent without question the true character of the curves. The D'Arsonval galvanometer was only slightly below the Panzer galvanometer previously used in working sensibility, but was vastly more steady in its action. Its resistance, and critical damping were each about 12 ohms, its sensibility 324 megohms, or 14.5 mm. per microvolt. The period was 7.3 sec. Adjustment for equal energy was made with a thermopile in just the same manner as in our previous work. Again the question whether equal energy falling upon these brilliantly reflecting crystals means equal absorbed energy among the various wave-lengths must be held in abeyance until some of their optical properties can be determined.

[1] PHYS. REV., 4, 1914, p. 507, 1914.

[2] Loc. cit.

The curves published in the former paper, to which this paper is an addendum, indicated in every case that the light sensibility was increasing as one proceeded below wave-length .50 μ. The indications pointed not only to an increase, but to a pretty sharp increase in sensibility. Our present work was done in the hope of locating a maximum in this region, but as far as we have gone the curves still rise, indicating the maximum, if there be any at all to be further into the ultra-violet. In the accompanying figure will be noted characteristic curves for two types of

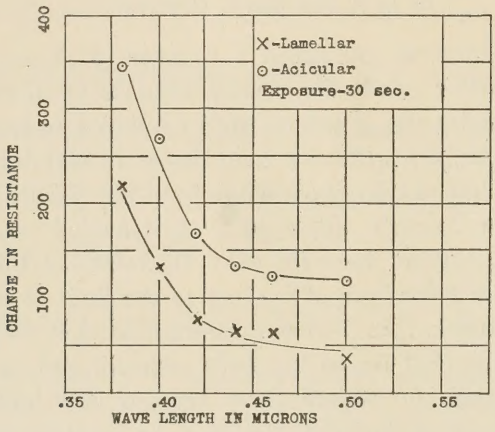

Fig. 1.

crystals, the lamellar and the acicular.[1] The former crystal was clamped between two conducting jaws and its thin edge was illuminated. The other crystal was similarly clamped, and was illuminated along its whole length a distance of about 7 mm. In both cases the action was what we called "direct," as distinguished from "transmitted" action. In other words, in this case, the light fell upon the conducting portion of the crystal. It should be noted that the energy used in these last experiments was about ten times as large as the energy employed in the work already referred to.

While it is unwise to make too close a comparison between the results from these crystals and those of work done on ordinary selenium cells,[2] it is interesting to note that although many of the characteristics are similar particularly with respect to light of the longer wave-lengths, there seems to be more of a difference when one deals with the shorter wave-lengths. For example we have thus far found no crystal that did

[1] For descriptions of these crystals, and for other references, see the paper previous to this: loc. cit. Also PHYS. REV. N. S., Vol. 4, p. 85, 1914.

[2] PHYS. REV., 4, 1914, p. 48.

not indicate an increase of sensibility toward the ultra-violet, whereas many of the cells tested showed a falling off in sensibility in this region. Nicholson[1] has carried some ordinary cells down to wave-length .23 μ, and has found a steady decrease in sensibility with the shorter-wave-lengths. As we noted above, however, there is no great value in making too close comparisons, because the peculiar mat, grey surface of the common selenium cell may react toward light in a manner decidedly different from that of these brilliant crystals.

It is interesting, even at the cost of going over old ground, to speculate upon what is going on in these crystals under light action. The increase in electric action toward the ultra-violet suggests a photo-electric effect. If that is the case then it becomes difficult to account for the maximum that appears in various places toward the red end of the spectrum. In the broad sense it is of course a photo-electric effect, only in this case it is not limited to the surface but takes place throughout the body of the crystal. And it must be remembered that it takes place through the crystal not only in the path of the light, but the action spreads to a distance, as we have already shown,[2] even going from the tip of a spine down into and through the main stem of a crystal cluster. If we are dealing here with the ordinary photo-electric effect, then we should find, when we proceed into the ultra-violet, no maximum whatever, but rather a steady increase in sensibility. It may be that there is a fundamental curve for the light-electric effect on the basis of Planck's theory which will show a steady increase in proceeding from the long to the short wave-lengths, and that the various maxima found in between will find their explanation in the common optical properties of these crystals, such as selective absorption, selective transmission, body color, etc. The fact that these peculiar maxima have thus far made their appearance in the red region of the spectrum, coupled with the fact that the color transmitted through these crystals is reddish in color may be of importance in the final solution. Work along this line is to be prosecuted at once.

Physical Laboratory,
State University of Iowa.

[1] Phys. Rev., 3, 1914, p. 1.
[2] Phil. Mag., 28, 1914, p. 497.

Digitized by the Internet Archive
in 2008 with funding from
Microsoft Corporation

http://www.archive.org/details/studiesinphysfir08iowauoft

[Reprinted from the PHYSICAL REVIEW, N.S., Vol. V, No. 1, January, 1915.]

NOTES ON ELECTRODE AND DIFFUSION POTENTIALS.

BY G. W. MOFFITT.

IN a recent paper[1] the writer has shown that the differences of potential between liquid surfaces and the air above them may be considerably greater than had usually been supposed. In some cases the value of this contact potential difference was changed as much as three tenths of a volt by slightly contaminating the surface of the liquid. It was also found that the value of this contact potential difference for clean fresh liquid surfaces is a steady value,—at least within a few thousandths of a volt,—and it is probable that the small variations in the observed values are due to the difficulties entering into the experiment.

It seems quite reasonable to assume then that there exists at the clean bounding surface of a liquid and air a definite difference of potential. The value of this potential in certain cases may be zero.

J. J. Thomson's work[2] on the electricity of drops points to the same conclusion from entirely different experimental evidence, as does some recent work by McTaggart.[3] Thomson found that certain solutio ıs—zinc chloride in water for example—in contact with air seem to show no electrification after the concentration has reached a certain value. McTaggart observed the motion of small air bubbles through liquids in which an electrostatic field was maintained. The bubbles were held in the axis of a horizontal glass tube filled with the liquid by the rapid rotation of the tube on its axis. The bubbles were free to move with or against the field which was parallel to the axis of the tube. He points out that the electrification set free by a falling drop is less than the charge as computed from the surface density and the area of the drop surface, and considers this last charge as the one effective in static, cataphoresis experiments. This means that only a part of the charge on a drop is set free by being broken up in air or by striking an obstacle. This would be expected. We would expect, however, that a drop neutral in the static condition would be neutral also when falling or when striking an obstacle. For the purposes of the present paper we may neglect any possible effects due to differences of concentration in the

[1] PHYS. REV., N. S., Vol. II., No. 2, Aug., 1913.

[2] Phil. Mag., Ser. V., Vol. 37. p. 341, 1894.

[3] Phil. Mag., Ser. VI., Vol 27. p. 297. 1914

surface film for results referred to later show that the neutral condition of the drop prevails throughout great variation of concentration. These views are supported by the few instances where data by the two methods can be compared. It may also be pointed out that in the cataphoresis experiments the liquid surface is sharply concave while with splashing drops it is either irregular or sharply convex. In view of these facts it seems reasonable to assume that conditions of electrical neutrality are the same for drops as for a clean level surface of the same liquid, and that solutions may be selected by the falling drop method whose air-liquid potential differences are negligibly small.

In the writer's work mentioned above values of the diffusion potential plus the difference of the two air-liquid potential differences were given for various solutions. If the solutions used had been those found by Thomson to exhibit no electrification in air we should have had values of the diffusion potentials for those particular cases. We have then a method of measuring potential differences between solutions in certain cases which is independent of theoretical assumptions. For instance, the diffusion potentials of zinc chloride solutions may be measured for concentrations greater than those for which the solution pressure theory seems to hold,—as determined by measurement of the electromotive force of concentration cells. It is by no means evident that an electrode potential for an electrode in an aqueous solution of an electrode salt can be computed in many cases, and diffusion potentials determined in this way seem likely to be in error. Especially is this true for the more concentrated solutions. In the case where the dissolved substance does not contain the element of which the electrode is composed we are still more uncertain. It seems therefore, that a direct method, tedious and inaccurate as it may be, would lead to results worth while.

On looking over Thomson's data on the electrification due to the splashing of drops no results were found for the solutions used by the writer in the work referred to above. It was decided to set up an apparatus similar in principle to that used by Thomson, and study the air-liquid surface potential differences for these solutions. A sensitive Dolazalek electrometer was used to detect the presence of the charges. One pair of quadrants was earthed, and the other was connected by means of a small copper wire to the solution in an insulated glass funnel from which the drops fell on a brass plate elevated from the bottom of a brass vessel in which the solution accumulated after splashing. This brass vessel was also insulated from the earth and connected to the insulated quadrants. Instead of using a fan to remove the charged air from the vicinity of the splashing drops a piece of brass tubing, flattened at one

end somewhat like a fishtail burner, was placed in a horizontal position on a line just above the plate on which the drops fell, and a few inches from the side of the brass vessel. This tube was connected to the compressed air supply and a gentle, steady stream of air kept passing over the vessel. The apparatus was frequently tested during the experiments for leakage and spurious charging effects by placing everything in position and observing the action of the electrometer when the solution was not dripping from the funnel. At times a slight positive charging was noted and wherever it appeared correction has been made for it in the results given below.

Distilled water was tried and a positive charge accumulated quite rapidly. A normal solution of sodium chloride was next tried and no charging resulted. Another portion of the solution was watered up to 0.5 normal. This produced no charging effect. Nor did a 0.1 normal solution prepared in the same way. A little distilled water was then run through the funnel to clean it and then the positive charging due to water was again observed. This is an example of the procedure in all cases distilled water being used before and after each series of solutions. In a similar manner solutions of sodium bromide, sodium sulphate, copper sulphate, and copper chloride were tried. All these were neutral in air at concentrations ranging from normal to less than tenth normal. Since the object of the experiment was to determine whether these solutions were neutral to air throughout this range of concentration no pains were taken to accurately measure the concentration.

These results show that the values published by the writer in PHYSICAL REVIEW, page 106, 1913, are, except in the case of zero concentration, the sum of the constant plate surface potential plus the liquid-metal potential difference in each case, and that the results on page 107 are the values of the diffusion potential differences for the solutions named.

Further, the effect of the anion on an electrode potential has been the subject of considerable discussion. It is generally looked upon as having little to do with the value of an electrode potential in the case where the cation is the same as the electrode. The variations in the two $ML + K$ curves for copper sulphate are probably due to uncontrolled factors. The two curves for sodium chloride also show some irregularities. The curve for sodium bromide lies close to the chloride curves, indicating that there is not much difference in the effects of these two anions on the electrode potential. The sodium sulphate curve, however, is removed from the other sodium curves, indicating that the sulphate ion has a different effect on the electrode potential than the halogen ions have. In these examples the cation is not the same as the electrode.

Why the anion should not affect the electrode potential is not clear. It seems more reasonable in the light of such cases as that just mentioned, to assume that in general the anion probably does exert some influence and that for the dilute solutions employed in concentration cells the effect at one electrode may be very nearly balanced by the oppositely directed effect at the other.

State University of Iowa,
September, 1914.

[Reprinted from the PHYSICAL REVIEW, N.S., Vol. V, No. 2, February, 1915.]

THE ELECTRICAL, THE PHOTO-ELECTRICAL AND THE ELECTRO-MECHANICAL PROPERTIES OF CERTAIN CRYSTALS OF METALLIC SELENIUM, WITH CERTAIN APPLICATIONS TO CRYSTAL STRUCTURE.

BY F. C. BROWN.

OUR information concerning the structure of the atom has perhaps advanced faster of recent years than has our information about the larger unit which is composed of atoms. The phenomena of radioactivity which are fundamentally independent of crystal structure have in a large measure furnished the data for studies on the atom. In the end the facts about either unit will aid in the understanding of the other unit of matter. Bragg's studies[1] on the reflection of X-rays show the crystal structure to be made up of stationary parts. These parts indicate charges of electricity resting almost in a plane. I wish in this paper to correlate some notions about the atom and the crystal after I have related some experiments with crystals of metallic selenium which point toward a new departure as to the rôle of the conducting electron in matter. These crystals of selenium show many unique properties involving co-related phenomena of electrical, optical and mechanical nature, and it is because of these new phenomena that we have a possible opportunity of arriving at further advances in the electrical view of matter. There will be given reasons however for believing that these related phenomena are merely accentuated in selenium much as the magnetic properties are accentuated in iron.

ACTION AT A DISTANCE BY LIGHT.

Recently[2] it was shown that when light illuminated one part of a crystal that there was a consequent change of electrical conductivity throughout the crystal. The electrical effect was observed in one case 10 mm. away from the point of illumination, and the effect was apparently as large as if the illumination fell on a point only 0.5 mm. away. In the latter paper referred to it was shown that this electrical effect could even be transmitted from one crystal to another when the crystals

[1] W. L. Bragg, Proc. Roy. Soc., *A*, 89, p. 248, and W. H. Bragg, Proc. Roy. Soc., *A*, 89, p. 277, 1914.

[2] Phil. Mag., Ser. VI, vol. 28, p. 497, and PHYS. REV., N. S., vol. 4, pp. 85 and 507, 1914.

were grown together. The essential difference between the direct action and the transmitted action as thus far observed is that the maximum effect for a given energy intensity is produced at longer wavelengths in the latter case. This shifting of the maximum sensibility was observed for 30-second exposures, which duration probably gave very nearly the equilibrium effect for a given intensity.

The action was, so far as could be observed, just as rapid when transmitted to a distance of 10 mm. as when transmitted only 0.5 mm. It thus can not be a mere temperature disturbance. Furthermore the major portion of the recovery after removing the light source was almost instantaneous.

When the crystal examined was illuminated at various points by a narrow beam of light it appeared that the light action was not uniform along the crystal. There were centers of varying sensibility. Thus the crystal has a mechanism of rather large dimensions, which when acted upon at different places produce results differing in magnitude.

The question that first arises is whether the equilibrium conductivity with a given illumination on a crystal represents an altered state of the crystal structure or whether it represents merely a condition in the crystal in which there is a constant liberation and supply of electrons that scatter throughout the crystal; the supposition being that for equilibrium the balance is kept up by the absorption of a similar quantity of electrons.

The Pressure Effect is not Transmitted.

It has already been shown[1] that the conductivity of either the acicular or lamellar crystals may be increased several hundred times by the application of mechanical pressure. Also it was demonstrated in the same paper that the absolute change of conductivity by a given intensity of illumination increased proportional to the conductivity in the dark. Apparently the increased pressure on the crystal made it easier for the light to free the electrons. The view that is here being taken is that the greater the pressure the greater is the number of electrons existing in a state of equilibrium almost unstable. When in the dark it is these semi-fixed electrons that are transferred from center to center by electrical potential differences and it is also these that light acts upon and makes free of the atomic structure. Consequently the greater the number of these semi-stable electrons the greater will be the change of conductivity by a given illumination. This is a fairly simple explanation of the increased light-sensitiveness of the crystals when under high pressures. Of course later information may make this explanation purely a conventional one.

[1] PHYS. REV., Ser. 2, Vol. 4, p. 85, 1914.

The above view was formulated to accord with the experimental result showing the pressure effect not to be transmitted to parts of the crystal not under pressure. The experiments were carried out as shown diagramatically in Fig. 1. The opposite ends, marked (1) and (2), of either a lamellar or an acicular crystal were placed between separate electrodes of brass. This apparatus permitted a number of experiments of varying character to be performed. If end (2) were illuminated the conductivity at end (1) changed almost as much as it did at the illuminated end. If the pressure were increased on end (2) the absolute sensibility to light increased almost proportional to the pressure, but the change of conductivity at (1) by illumination at (2) was not increased by this increase of pressure on end (2). However if the pressure were applied at (1) instead of (2), and the illumination just as above on end (1), then there was an increase in the absolute conductivity at (1).

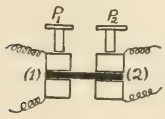

Fig. 1.

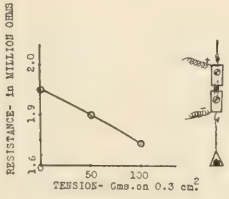

Fig. 2.

The conclusion is that the *pressure effect merely makes it easier for the light to change the conductivity and that the pressure does not act except at the point of application.* As will be pointed out elsewhere these results lead directly to the conclusion that pressure does not increase the conductivity by adding free electrons. And in view of the fact that the increased light-sensitivity due to high pressure is not transmitted we are justified in concluding that light does not produce free electrons in the generally accepted sense. In other words none of the conductivity in these crystals can arise from free electrons such as exist in metals according to the hypothesis of Richardson and Brown.

In the above experiment it was immaterial whether or not a current was flowing across both ends of the crystal simultaneously. A second set of experiments was made, with the same apparatus in such a way that the current flowed all the time through the part of the crystal under study and this same part of the crystal was not under pressure by the electrodes.

The pressure was applied simultaneously on both ends of the crystal. The essential part of the resistance was between the electrodes and this part of the crystal was obviously not under pressure. By varying the

pressure on the crystal there was no change in the resistance. The electrodes were separated in different tests by distances ranging from 0.5 to 5 mm. In no case was there evidence that the change of resistance by pressure extended beyond the region under stress. Likewise the light-sensitiveness of the middle portion of the crystal did not increase as a result of the pressure on the ends.

Another related electro-mechanical effect is the change of resistance accompanying a stretching force. For the study of this effect five branch crystals growing out from a central spine such as reproduced in the earlier article were chosen. The opposite ends of these crystals were clamped in brass electrodes as shown in Fig. 2, so that the stress was distributed among the five crystals. The crystals were stretched by adding weights to a pan pulling on one of the electrodes as shown. This experiment was rather difficult because the apparent malleability of the crystals caused them to flatten out, tear and pull out of the clamps and also because the slightest twisting would cause the crystals to weaken and break. However I did succeed in observing that the crystals would withstand a stretching force greater than 10 kgm. per square centimeter. With such stresses there was a decided decrease of resistance as shown in the curve of Fig. 2. When the weights were removed the resistance usually increased again to its previous higher value without stress, thus indicating the crystal to be in equilibrium either with or without the additional forces.

The change of resistance for tensile forces does not seem to be as great as for compression forces as previously related. The interpretation that is to be placed on these results is that stress by stretching brings an increased number of electrons into almost unstable equilibrium and thus increases the current with a given potential difference. The fact that the effect is not so large as by compression forces may arise from an increasing of the distance between some of the semi-stable electrons, when this distance is measured in the direction of current flow.

The Electrical Conductivity with Varying Electrical Stresses.

On the preceding view of the structure of the crystal in which conducting charges are tied up in the atom in a quasi-stable condition, we should expect the number of electrons that could be dragged out of their fixed positions would vary with the electrical forces acting on them, and that only forces acting in the direction of current flow would alter the magnitude of the current. In what way the current should vary with the electrical forces will depend upon a more exact picture than we are yet able to formulate. When a lamellar crystal was under pressures only

slightly greater than atmospheric, the resistance varied with increasing potential difference at the electrodes as shown in the lower curve of Fig. 3. The potential was applied in the direction of the current flow for about 20 seconds. For low potentials the change of resistance was almost steady after this time, but for high potentials, above 100 volts, there were signs of unsteadiness if the current were left on too long. Next the pressure was increased so that the conductivity increased twenty times in the dark, and the resistance was then observed to vary as shown by the upper curve in Fig. 3. At first sight one might be inclined to say that the change of resistance with varying electro-motive force is materially less with high pressure on the crystal. But inspection shows that only the percentage change is greater with low pressure. The following table gives the conductivity for some potential differences as deduced from the data graphed in Fig. 3.

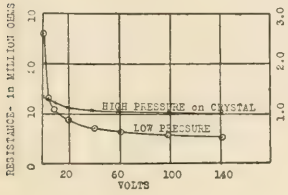

Fig. 3.

Difference of Potential.	Conductivity in Dark.	
	With Low Pressure.	With High Pressure.
1.4 volts	3.84×10^{-8}	75.2×10^{-8}
10 volts	9.17×10^{-8}	83.3×10^{-8}
41 volts	14.3×10^{-8}	96.1×10^{-8}
100 volts	18.2×10^{-8}	98.0×10^{-8}
143 volts	19.6×10^{-8}	100×10^{-8}
Extreme variation in absolute specific conductivity	16×10^{-8}	25×10^{-8}

It is observed that the extreme variation of conductivity for the potentials used was somewhat greater with the crystal under high pressure than under low pressure. This merely signifies that the saturation current was not nearly reached by increasing the conductivity by a factor of twenty. The increased pressure probably makes the electrons free to leave the atomic structure with lower potentials. The instability of the electrons is increased by either mechanical pressure or electrical stresses.

Light-sensitiveness with Different Potentials.—If light produces free electrons and the electrical stress merely pulls electrons out of the atomic structures in the line of conduction, we should expect that the conductivity by illumination would be increased by the same amount regardless of the potentials across the crystal. This is on the supposition that the conductivity increases proportional to the increase in the

number of conducting electrons, and that the stability of the fixed electrons in no wise determines how many are to be freed by light.

The following table gives the results that were obtained to check the validity of the above, when the crystal was illuminated with light of constant intensity until equilibrium was reached.

Potential Fall Across Crystal.	Resistance.		Ratio of $\frac{C \text{ in Dark}}{C \text{ in Light}}$.
	In Dark.	Illuminated.	
1.4 volts	1.30×10^6	$.82\times10^6$	
	1.20 "	.80 "	
	1.20 "	.79 "	
	1.20 "	.79 "	
	1.20 "		1.53±.03
60 volts	$.475\times10^6$	$.299\times10^6$	
	.475 "	.310 "	
	.49 "	.323 "	
	.492 "	.325 "	
	.495 "	.330 "	1.53±.02

It is observed that the percentage increase of conductivity is the same regardless of what the initial conductivity may be as influenced by the potential fall across the crystal. But this means that the absolute increase of conductivity is nearly three times greater with the higher voltage. Therefore the presumptions mentioned above are not true. The exactness of the ratio of the increase of conductivity would favor the view that the light renders a constant number of electrons in a quasi-stable equilibrium, and that the apparent increased sensibility by using higher potentials is merely the result of a pulling out of a greater number of semi-stable electrons.

It should be noted that the constancy of the light-sensibility ratio above shown for electrical potentials is identical to the result noted in my previous paper[1] where the percentage increase of conductivity by illumination remained constant for varying pressures. No doubt the two sets of results have an analogous explanation and this leads to the suggestion that electrical stresses and mechanical pressures alter the equilibrium of the crystal structure in identical ways.

The Non-transmissibility of the Electrical Potential Effect.—If the above identity exists we should expect that the effect of high potentials should alter the conductivity only on that part of the crystal in the immediate field the same as found for the pressure effect. Two experiments were carried out in a fairly satisfactory manner to answer this question.

In the first experiment one end of an acicular crystal was placed

[1] Loc. cit.

between electrodes differing in potential by 1.4 volts. The exterior circuit was closed through a galvanometer as shown in Fig. 4. The deflection of the galvanometer was noted. Then a potential difference of 100 volts was applied across the opposite end (*B*) of the crystal. This high potential changed the conductivity of end (*B*) by about a factor of five, but there could not be detected the slightest change of conductivity at the end *A*, as result of the potential effect, even when the opposite sets of electrodes were approached within one millimeter of each other. It should also be noted that there was no permanent potential generated at (*A*) by the application of the high potential at (*B*). Such a potential would have been indicated by the galvanometer.

Fig. 4.

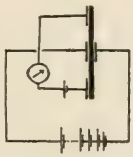

Fig. 5.

The second experiment was designed to detect any effect by an electrical stress acting at right angles to the current flow. The high potential was applied across the middle of the acicular crystal as shown in Fig. 5. The conductivity was being measured by a current flowing lengthwise of the crystal. However a strip of mica on one of the middle electrodes prevented the flow of a current by the high potential of 200 volts.

The result of this experiment was also negative. These two experiments show that a selenium crystal may change its resistance along one axis without altering the resistance along a perpendicular axis. This result is explicable on the view that free electrons are not the current bearers in non-illuminated selenium.

It may be that the piezo-electric effect, as exhibited for example in quartz where a number of electrons are freed by pressure, bears a certain resemblance to the effect described above. The quartz perhaps does not show a corresponding change of resistance because of its extreme insulating properties.

Conclusions.

It has been shown, (1) when a selenium crystal is illuminated at certain points that the conductivity of the entire crystal is increased, (2) that when pressure is applied to the crystal only that part of the crystal under pressure is altered, (3) that electrical forces alter the conductivity only of that part directly under the forces and further that the influence is exerted only in the direction of the electrical force.

The direct conclusion is that light action has to do with an essentially different mechanism than electrical stresses or mechanical stresses. Starting with the above fundamental facts and correlating the other facts mentioned in the paper, I have attempted to formulate certain notions about the structure of the crystal.

This notion premises that a crystal when in the dark has no free electrons in the ordinary sense such as was found to exist in certain metals by the hypothesis and experimental work of Richardson and Brown.[1] True a crystal conducts electricity when in the dark, but this conductivity is small compared with that of the metals. The elementary notion of the crystal is merely a structure composed of positive and negative charges in equilibrium with each other. But this equilibrium is for a large number of the electrons at least in a very low degree of stability. The electrons would be held in equilibrium by the positive forces essentially, but certain of them while necessary to the complete atomic structure, would nevertheless leave the centers (*i. e.*, perhaps atoms) when under small stresses. So long as an electron remains outside an atom requiring one or more electrons, this electron would behave as the traditional free electron. Thus whatever makes free electrons would increase the conductivity.

The hypothesis is that the conductivity in the dark does not arise from the free electrons, except those that have not had time to adjust themselves following an internal disturbance, but from electrons that are pulled from one atom to the neighboring atom and so on by the electrical forces across the crystal. The following is evidence for this view; first it was noted that for very small electromotive forces the resistance was almost infinite and for increasing potentials up to a certain limit the resistance decreased very rapidly. Secondly, it was noted that very large potential differences acting at right angles to the current flow did not alter the magnitude of the current.

The pressure effect is readily explicable on the basis that no free electrons exist in the crystal when in the dark. Pressure may increase the conductivity many hundred fold, but it will not influence the resistance outside the part of the crystal pressed upon. It would seem then that mechanical pressure merely pushes the electrical charges, associated with neighboring atoms, into a less stable equilibrium, perhaps closer proximity, so that a given electrical stress can pull more electrons from one atom to the next.

We are now in a position to assert something concerning the nature of light-action, based on the fundamental property of transmitted action.

[1] Phil. Mag., Ser. VI, 16, p. 353, 1908.

By some mechanism the light can, no doubt because of its electromagnetic properties, lower the degree of stability of many or all of the electrons throughout the crystal. These same electrons may have their degree of stability yet further lowered by mechanical pressure. The lower the average stability of the electrons the greater will be the current with a given potential difference. With a greater potential difference the same light intensity would therefore seem to produce a greater change of conductivity. The electrons after removal from their fixed positions may behave, until reunited in the structure, somewhat as the traditional electron.

The action of light is not local. The electrons are made less stable or temporarily free by an indirect mechanism operating everywhere in the crystal. The effect is almost uniform at all points. It travels too fast to be a temperature transmission and the maximum sensibility for the transmitted action is in the visible spectrum. The transmission is at least analogous to that of a mechanical vibration, although only certain parts of the crystal may take part in its operation.

The reader will observe that aside from the experimental work the essential new thought in this paper is a new hypothesis to explain the nature of electrical conduction in certain crystals. This hypothesis bears some resemblance to the accepted theory of electrolytic conduction, the distinctive feature being that only electrons move from one center to the next in the chain of centers between electrodes. This view requires that the crystal shall have fixed electrons in its structure but no permanently free electrons.

The Physical Laboratory,
The State University of Iowa.

Fig. 1.

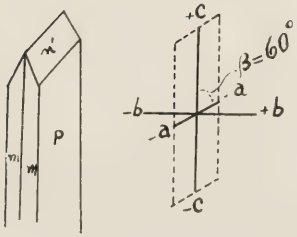

Fig. 2.

F. C. BROWN.

[Reprinted from the PHYSICAL REVIEW, N.S., Vol. V, No. 3, March, 1915.]

ISOLATED CRYSTALS OF SELENIUM OF THE SECOND AND FIFTH SYSTEMS, AND THE PHYSICAL CONDITIONS DETERMINING THEIR PRODUCTION.

BY F. C. BROWN.

RECENTLY[1] I described new crystal forms of metallic selenium, in connection with their optical and photo-electrical properties. One form was definitely classified as belonging to the rhombohedral hexagonal system, but because of the smallness and incompleteness of the lamellar crystals they could not be classified. Since then we have succeeded in obtaining lamellar crystals well developed on all surfaces, so that the angles between all faces could be determined with a certain accuracy, by the use of a circle goniometer.

The reproduction in Fig. 1 is that of lamellar crystals belonging to the fifth or monoclinic system. A diagram of one of the crystals is given in Fig. 2, together with the plane of symmetry and the axes. The surfaces m, n and O are at right angles. The ratios of the axes of one crystal were about $a : \bar{b} : c = 1.0 : 0.18 : 6.0$, $\beta = 60°$. The longest crystal was about 10 mm.

Crystals of the hexagonal system were photographed and reproduced in the previous article referred to. In the acicular crystals the angles between the faces were all 60°. Other crystal forms and growths were reproduced but thus far it has not been possible to definitely classify them.

Crystals of both systems were produced by sublimating amorphous selenium after it had been melted. In general the higher the temperature at which the crystals were formed from the vapor, the more certainly would the hexagonal needles form. At temperatures between 190° and 220° large quantities of the hexagonal needles were obtained, and at the highest temperature various undetermined forms and combinations of forms appeared. It required one or two weeks for the crystals to form in the larger sizes.

By keeping the oven temperature at 170° ± 1° for a period of three months the monoclinic crystals shown in Fig. 1 were made. The selenium vapor was driven from the amorphous form at a somewhat higher temperature and condensed on the crystal at the above temperature. These

[1] PHYS. REV., N. S., Vol. 4, p. 85, 1914.

crystals were formed in a vacuum, except for the selenium vapor present. It was observed that these lamellar crystals would also form at temperatures as high as 190° in an atmosphere of air at normal pressure. Also it seemed necessary to increase the temperature necessary for the formation of the hexagonal form when the vapor was deposited in air at normal pressure. These observations lead to the supposition that the vapor density as well as the temperature is a determining factor in the control of the crystal forms.

As far as has been tested the electrical, the light-electric, and the electro-mechanical properties of the selenium crystals of the two systems do not show any distinct differences. The value of the conductivity, and the sensibility to light in different parts of the spectrum are of the same order of magnitude. The above facts indicate that the above properties are seated in the molecular structure, and that the minor variations are controlled by the crystal structure.

It has been observed that the light on going through the wedge-shaped end of a lamellar crystal is deviated about 30°. Since we know the angle of the wedge, we have a direct method of measuring the index of refraction of a conducting substance. This will be carried out at the earliest opportunity.[1]

UNIVERSITY OF IOWA.

[1] Since writing this article I have obtained the red crystalline variety of selenium by sublimation. These crystals are doubly refracting and are no doubt the same as those obtained by various chemical means. The precise physical conditions *i. e.*, vapor pressure and temperature governing the formation of the red crystals by sublimation are yet to be determined.

[Reprinted from the PHYSICAL REVIEW, N.S., Vol. V, No. 5, May, 1915.]

THE CHANGE IN THE ELASTICITY OF A MILD STEEL WIRE WITH CURRENT AND EXTERNAL HEATING.

BY H. L. DODGE.

THE effect of current and external heating upon the Young's modulus of a copper wire was described in a previous number of the PHYSICAL REVIEW.[1] The present paper deals with a similar investigation of a mild steel wire, the more marked changes being an increased temperature range, namely 20° C. to 475° C., and slightly greater accuracy. For a detailed description of the apparatus and method of making the determinations the former paper should be consulted.

IMPROVEMENTS IN APPARATUS AND METHOD.

Mention cannot however be omitted of certain changes which have resulted in greater accuracy. Attention was called in the previous paper to the desirability of having the specimen of wire free from mechanical interference between the two points of suspension. The only interference in the case of copper wire was the three thermo-couples of fine wire which did not seem to cause any inaccuracy in the results. Some changes in the method of measuring the temperature have made these unnecessary and in the present apparatus the specimen of wire hangs absolutely free.

As before, the temperature of the wire is determined by its change in length with increase of temperature. In the former paper the justification and advantages of this method were explained. The same statements apply in the present instance as the only changes have been in the process of finding the relation between temperature and length. This was found as follows:

A thermo-couple was carefully calibrated by means of a thermometer. It was then attached to the wire at the middle point and the relation between the temperature of this point and the length determined. As all the points of the wire were not at the same temperature it was necessary to find the relation between the average temperature and the temperature of the middle point. The heating box was maintained at a constant temperature of about 350° C. and a large number of thermo-couple readings taken at various points on the wire. This not only

[1] PHYS. REV., N. S., Vol. 2, 431, 1913.

established the relation between the average temperature of the wire and its length but also gave complete information concerning the temperature distribution. Under the above conditions the maximum variation of the temperature along the wire was about 40° C. while the error in the measurement of the average temperature was probably not more than 10° C. At lower temperatures the accuracy is of course greater, the error amounting to but one or two degrees at or near room temperature and perhaps five degrees at 150° C. At higher temperatures the possibility of error is of course greater amounting to about 20° C. at 475° C. The curve representing the relation between change of length and temperature was plotted and found to be a straight line within the limits of accuracy of the temperature measurements. The increase in length between 20° C. and 475° C., corrected for the stretch caused by the decrease in the modulus, was 3.67 mm. As the wire was 57.6 cm. between points of observation the mean temperature coefficient of expansion proves to be .000014 with a possible error of five per cent.

Considerably increased accuracy has been secured in the measurement of the stretch and consequently the modulus with the present apparatus by the use of new microscopes and micrometer slides with screws accurate to .001 mm.

The Specimen.

The sample of wire upon which the tests were made was obtained from the Driver-Harris Wire Co., Harrison, N. J. A chemical analysis of the wire secured from Mr. J. W. Whitfield of the firm of Booth, Garrett and Blair, Philadelphia, Pa., was as follows: Silicon, 0.057 per cent.; sulphur, 0.059 per cent.; phosphorus, 0.105 per cent.; manganese, 0.735 per cent.; carbon, 0.162 per cent.; iron (by differences), 98.882 per cent. The sample was pronounced a " mild " or low carbon steel.

The mean coefficient of expansion between 20° C. and 475° C. is 14×10^{-6} per degree Centigrade to an accuracy of about five per cent. The wire is .82 mm. in diameter. The portion between the points of observation was 57.6 cm. long. A weight of 2,109 grams was kept upon the wire continually, the modulus being determined by measuring the elongation produced by an additional weight of 2,252 grams. Certain additional facts regarding the loading, current, number of readings, time between readings, duration of tests, etc., appear in Table I. which contains the complete data of a characteristic series of observations with current heating. The other series differ materially only in method of heating or temperature range.

Nature of the Tests.

The preliminary observations made while the wire was being straightened and brought to a cyclic state revealed striking irregularities in its behavior. At first increase of temperature increased the modulus but this effect soon disappeared. Continued heating and testing gradually increased the modulus. This was probably due largely to stretching and straightening. Both current and external heating were employed and no effect characteristic of the method of heating could be detected. After a number of cycles of heating and cooling accompanied by stretching, the wire was brought to a cyclic state and the more extended tests made.

The first group of observations were with a temperature range of 20° C. to 300° C. The tests were with both external and internal heating and increasing and decreasing temperature and were so arranged as to reveal if possible any effects peculiar to the method of heating or the thermal route by which any temperature was reached. Later the temperature range was extended to 450° C. and to 475° C. and similar series of tests made. A few characteristic results are shown in Figs. 1, 2 and 3, explanations of which follow.

Explanations of the Curves.

Figs. 1, 2 and 3 represent graphically certain of the results that have been obtained. Table I. contains the complete data for series 19, Fig. 2. The data for the other curves do not differ materially in general character. In every case the dotted points represent determinations of the modulus with increasing temperature and the crosses those with decreasing temperature. Nearly all the points are the result of ten or more observations. When the terms "external" and "internal" heating are employed the former indicates that the wire was heated by means of a heating element of nichrome wire extending along the bottom of the enclosing box, while the latter term refers to heating by means of an electric current passing through the specimen itself.

In all of the statements regarding the absence of certain effects it should be understood that this refers to effects amounting to one per cent. or more. The present apparatus cannot detect variations in the modulus of a less amount. The accuracy of the work can best be judged from the curves. A change in the modulus of as much as ten per cent. corresponds to a difference in stretch of only 0.013 mm.

The first four series covered a temperature range of 20° C. to 300° C. with both external and internal heating and showed a practically linear relation between Young's modulus and temperature. Series 5, Fig. 1,

was with external heating, increasing and decreasing temperature. Series 6 and 8 are plotted together. They were with internal heating, increasing and decreasing temperature. The results of the first eight series show that for temperatures below 300° C. the elasticity is a definite function of temperature, independent of manner of heating and free from any hysteresis effects.

It was next thought desirable to extend the temperature range by a considerable amount and to repeat the tests in various orders to ascertain whether the same relations hold for a greater range of temperature. Upon heating above 300° C. a very rapid decrease of the modulus was discovered, the rate of decrease becoming very marked at about 450° C. The results are shown in Fig. 1, series 9. At this temperature the wire

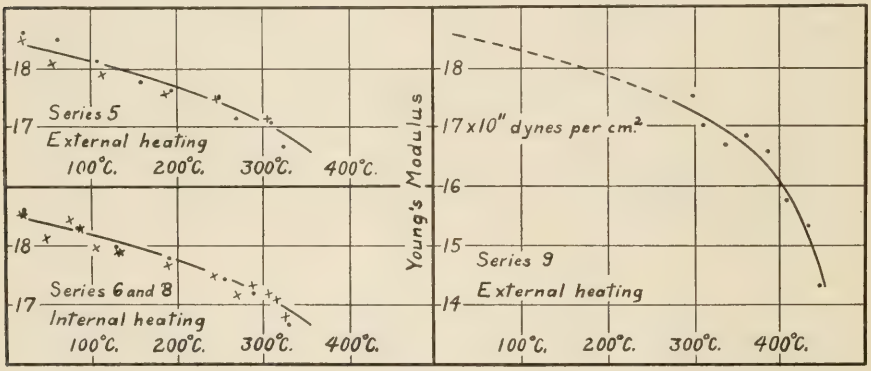

Fig. 1.

Change of Young's modulus of a mild steel wire with temperature.

stretched about one fourth of a millimeter, approximately .05 per cent. of its length. When the elastic limit is exceeded there should be an increase of the modulus for all temperatures. The next observation at room temperature showed an increase from 18.5 to 19.2 or 2.6 per cent. The increase of the modulus for other temperatures as well can be seen from Fig. 2.

The first three series of observations of Fig. 2 give the temperature variation of Young's modulus with external heating. Series 11 shows the type of curve, and the return of the modulus to the same value after a day of heating and stretching. Series 12 was taken under similar conditions. By yielding the same results it proves the total disappearance of all history effects except the very gradual increase of the modulus with continued heating and stretching. As the observations for decreasing temperature in the last two series were few, series 13 was made in order to learn whether the curve with decreasing temperature is the same as

with increasing temperature. The results indicate the absence of any hysteresis effect. In order that the various series in this and the other figures may be more easily compared the curve which appears was drawn to fit a composite of them all. Its shape is exactly the same in every case

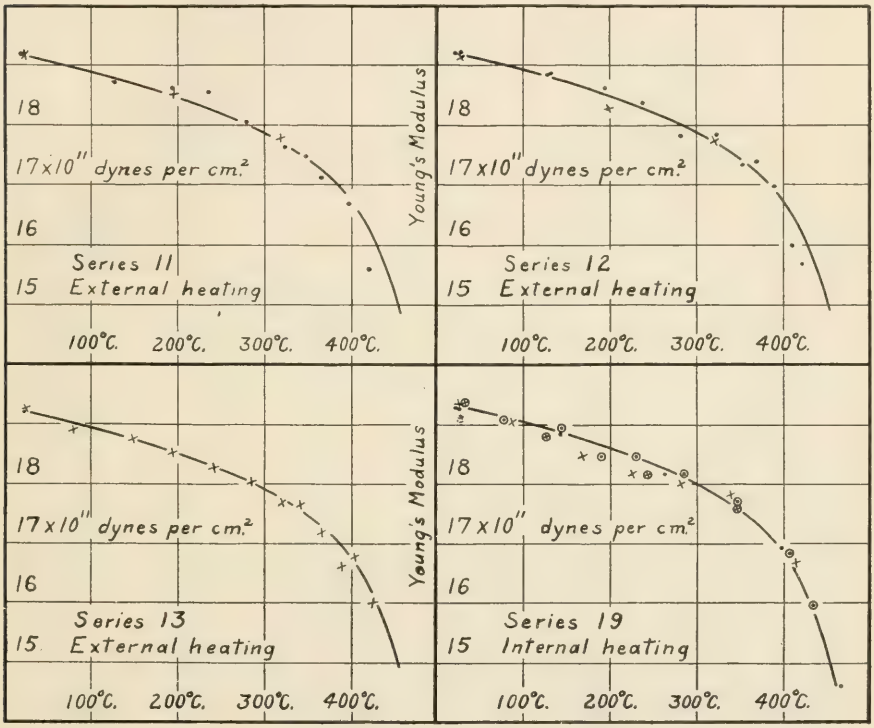

Fig. 2.

Change of Young's modulus of a mild steel wire with increasing and decreasing temperature and with internal and external heating.

the only difference being that it is gradually raised to correspond to the general increase of the modulus as the work progressed.

All of the observations so far had been made with the wire enclosed in an asbestos board heating box so that the currents used in the wire and the heating element might be as low as possible. Walker[1] has found a great difference between the effects of internal and external heating, dependent in part upon the current density. He interprets these results as caused by the magnetic effect of the current, the maximum field which he secures having an average value of 31 gausses.

When series 19 (see Table I.) was taken an attempt was made to find this effect in case it were associated with residual magnetism. Between

[1] Proc. Roy. Soc. Edin., Vol. 31, 186, 1910.

the first two observations of this series (Fig. 2) the wire was subjected for an instant to a current corresponding to an average field of 50 gausses. The difference in the two plotted points is but a fraction of the experi-

TABLE I.

Data for Series 19, Internal Heating.

Length of wire ..... 57.6 cm.
Diameter of wire ..... .82 mm.
Unvarying load ..... 2109 g.
Added load ..... 2252 g.
Total load per sq. mm. ..... 8.2 kg.

Obs. No.	Time.	Temp.	Stretch.	Current.	No. of Obs.	Young's Modulus $\times 10^{-11}$ Dynes per Cm.^{2}
1	9:30 A.M.	22° C.	.1250 mm.	0.0 amp.	12	19.3
2	10:00	24	.1250	0.0	10	19.3
3	10:20	142	.1280	5.7	10	18.9
4	10:55	263	.1325	7.4	10	18.2
5	11:35	398	.1425	8.5	11	16.9
6	11:50	466	.1650	9.0	5	14.6
7	12:50 P.M.	416	.1445	8.5	10	16.7
8	1:20	340	.1350	7.9	12	17.8
9	2:00	282	.1340	7.4	11	18.0
10	2:45	224	.1325	6.8	11	18.2
11	3:30	168	.1305	6.0	10	18.5
12	4:00	86	.1265	4.0	10	19.1
13	5:00	26	.1240	0.0	11	19.4
14	5:25	77	.1265	3.8	10	19.1
15	5:50	144	.1270	5.6	11	19.0
16	6:50	188	.1305	6.4	10	18.5
17	7:30	230	.1305	7.0	10	18.5
18	7:55	284	.1325	7.6	11	18.2
19	8:20	384	.1360	8.2	11	17.7
20	8:45	408	.1430	8.5	14	16.8
21	9:05	434	.1560	8.7	10	16.0
22	9:30	346	.1370	8.0	11	17.6
23	9:55	244	.1325	7.0	11	18.2
24	10:30	128	.1280	5.0	12	18.8
25	11:00	28	.1240	0.0	10	19.4

mental error. Neither this test nor any of the other results up to this time gave any suggestion of the effects Walker reports.

It seemed desirable however to employ as large currents as possible in an attempt to bring out the effect. Accordingly the cover of the box enclosing the wire was removed giving the air free access. Series 21, Fig. 3, is the result, the maximum current being 9 amperes, corresponding to a current density of 17 amperes per square millimeter and an average field of 29 gausses. For purposes of comparison series 22 was taken with

external heating, the slight tendency of the readings to fall lower (less than .001 mm. difference in the actual measurements) being probably due entirely to the experimental errors caused by the fact that the main

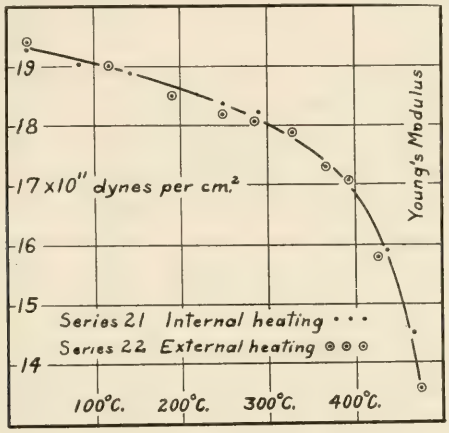

Fig. 3.

Results with internal heating, using the largest possible currents, compared with those with external heating.

part of the apparatus was much hotter in the latter case. No indication of the effects reported by Walker is to be found.

COMPARISON WITH THE RESULTS OF OTHERS.

The earlier work[1] upon the effect of an electric current upon Young's modulus is of very questionable value. Reference will be made only to the more recent work of Walker and Miss Noyes. Miss Noyes[2] found no effect peculiar to current in the case of piano wire but her work was not of sufficient accuracy to have revealed even a relatively large difference. Walker,[3] on the other hand, taking measurements with what appears to be a considerably greater accuracy found striking differences of large magnitude. There are several points concerning even his most recent work that make one seriously question whether the results which he secured are attributable to the causes to which they are assigned. A lengthy discussion of Walker's work is out of place at this time but two or three differences in our results may be of interest.

Walker worked over a temperature range of 20° C. to 125° C., finding in the case of iron wire a maximum value of the modulus at about 50° C.,

[1] Wertheim, Ann. de Chim. et de Phys., *12*, 610, 1844. Edlund, Annal d. Phys., *129*, 15, 1866; *131*, 337, 1867. Streintz, Annal d. Phys., *150*, 368, 1873. Mebius, Oefvers. af k. Vet.-Akad. Forhandl., 681, 1887; Beibl., *12*, 678, 1888.

[2] PHYS. REV., O. S., Vol. 2, 279, 1895; Vol. 3, 433, 1896.

[3] Proc. Roy. Soc. Edin., Vol. 27, 343, 1907; Vol. 28, 652, 1907; Vol. 31, 186, 1910.

the value at 125° C. being about the same as at 20° C. Employing different loads he finds that the maximum is less with the heavier loads amounting to 11.9 per cent. for 18.3 kg. per sq. mm. and only 1.4 per cent. for 32.9 kg. per sq. mm. With steel wire Walker obtains similar results except that at higher temperatures the modulus is a little less than at room temperature and the magnitude of the change in the modulus is smaller. With a load of 23.3 kg. per sq. mm. the maximum value occurs at about 60° C. and is 1.9 per cent. greater than the value at 15° C. For 42.4 kg. per sq. mm. the maximum is at about 35° C. and amounts to 0.3 per cent. The load which I have employed is only 8.2 kg. per sq. mm. It would seem as if some trace of Walker's maximum ought to have appeared, but in my results there has been no indication of any such effect.

Walker[1] also studied the effect of variation of load while keeping the current constant at several different values. For some reason the results are now interpreted in terms of field intensity produced in the wire although formerly the effect of the current was regarded as one of temperature. The fields employed varied from 0.7 to 31.9 gausses for iron and from 0.65 to 33.7 gausses for steel. The loads were 14.6 to 36.5 kg. per sq. mm. and 16.9 to 46.7 kg. per sq. mm. respectively. In each case maximum values of the modulus appeared with intermediate values of both load and field. In the case of iron the maximum variation in the modulus for a given field was 11 per cent., the field being 9.1 gausses and the load 18.3 kg. per sq. mm. The maximum variation for steel was 2.4 per cent. and occurred with a field of 17.4 gausses and a load of 29.7 kg. per sq. mm. As these effects were interpreted as results of magnetic field intensity it is worthy of remark that exactly the same sort of variations were found with the non-magnetic metals copper and platinum.

In my own work the average field intensity is 3.25 times the current in amperes. It varied therefore from 0 to 29 gausses and since the external heating was sometimes used in conjunction it was not always the same for the same temperature. In no instance have I been able to detect any effect peculiar to the current or its accompanying magnetic field and any variation in the modulus amounting to more than one per cent. would have been observed.

Concerning the effect of magnetization upon Young's modulus a great deal of work has been carried on. The more important results have been those of Honda and Terada,[2] Honda, Shimizu, and Kusakabe,[3] Rensing,[4]

[1] Proc. Roy. Soc. Edin., Vol. 31. 186, 1910.

[2] Phil. Mag., Vol. 13, 36, 1907.

[3] Phil. Mag., Vol. 4, 459, 1902.

[4] Annal. d. Phys., *14*, 363, 1904.

Stevens and Dorsey,[1] Stevens,[2] Brackett[3] and Bock.[4] An effect has been observed but in every case it was small, amounting to less than one half per cent. As far as these results, which are for longitudinal and not circular magnetization, and my own can be compared with those of Walker there is flat contradiction. In the near future I hope to have more data as a basis for comparison. The differences may of course be due entirely to an inherent difference in the samples of wire.

The results of the earlier investigations of the temperature coefficient of Young's modulus of iron and steel wires were over rather limited temperature ranges and were subject to considerable error. Wertheim[6] found for iron a maximum in the neighborhood of 100° C. followed by a rapid decrease up to 200° C. A similar maximum appeared in English steel but not in cast steel. Kupfer[6] employing a method of bending rods found for several metals including iron and steel a decrease of elasticity with increase of temperature. More recently Gray, Blyth, and Dunlop[7] have worked with mild steel and Shakespeare[8] with a "silver" steel wire at room temperature and at or about 100° C. and found a decrease in the modulus per 100° C. of 2.47 and 3.8 per cent. respectively [1.5]. For iron wire the coefficients are 1.6 and 1.8 per cent. respectively while Katzenelsohn[9] reports 2.33 per cent. The bracketed figures here and following are results taken from my own work for corresponding temperatures. The work of Miss Noyes[10] was not sufficiently accurate to have detected other than a linear relation had it existed. She finds for four samples of piano wire with moduli of about 20.3 an average decrease of the modulus of 4.6 per cent. per 100° C.

On the other hand Pisati[11] finds for both iron and steel a decrease of the modulus at an increasing rate for a temperature range extending to 300° C. His value of the modulus for steel is 18.47. Heating from 25° C. to 300° C. produces a decrease of 6.3 per cent. [19.3, 6.7]. For iron the modulus was 21.43, the decrease, 12 per cent. Walker[12] found the modulus of a piece of soft iron wire to be 18.22 at 17.5° C. With ordinary heating up to 129° C. there was a uniform decrease amounting to 3.6 per cent. The

[1] PHYS. REV., O. S., Vol. 9, 116, 1900.
[2] PHYS. REV., O. S., Vol. 11, 95, 1900.
[3] PHYS. REV., O. S., Vol. 5, 257, 1897.
[4] Annal. d. Phys., *54*, 442, 1895.
[5] Ann. de chim. et de Phys., *12*, 385, 1844; *15*, 114, 1845.
[6] History of Elasticity, Todhunter and Pearson, Vol. II, p. 519.
[7] Proc. Roy. Soc., Vol. 67, 180, 1900.
[8] Phil. Mag., Vol. 47, 539, 1899.
[9] Diss. Berlin, 1887.
[10] *Loc. cit.*
[11] Nuovo Cimento, *4*, 152, 1878.
[12] Proc. Roy. Soc. Edin., Vol. 28, 652, 1907.

modulus of a steel wire was 21.6 at 15° C. It showed a decrease of 2.5 per cent. at 129° C. [19.3, 2.1].

The results of Pisati,[1] Gray, Blyth, and Dunlop,[2] Kohlrausch and Loomis,[3] and Slotte[4] upon the torsion modulus of iron and steel are of interest showing as they do a decrease of the modulus with increase of temperature, the decrease in nearly every case becoming more rapid at the higher temperatures.

Discussion of Errors.

At the lower temperatures the most important source of error is the measurement of the modulus. For temperatures below 350° C. the influence of the temperature upon the modulus is so small that errors in temperature are negligible in comparison. Above 350° C. the temperature effect becomes greater and greater and the errors in the temperature are the more important.

Even with the measurement of the stretch made with micrometer slides accurate to 0.001 mm. it is impossible to be certain of the value of the modulus from any single determination below 350° C. to a greater accuracy than one per cent. In some cases the results vary as much as two per cent. At 350° C. a change in the temperature of 20° C. would be necessary to produce a change of one per cent. in the modulus. At this temperature the measurement of the average temperature is accurate to about 10° C. which is entirely sufficient for the purposes of this investigation.

Above 350° C. the error in the modulus becomes greater amounting to three or four per cent. at 475° C. Over this range the rate of change of the modulus with temperature is so rapid that the errors in the modulus are now negligible in comparison with the temperature error which is perhaps as great as 20° C. at 475° C.

The errors in the modulus are errors of observation and in the plotting of the curves tend to cancel each other. The change of length can be observed to an accuracy corresponding to a temperature difference of one eighth of a degree. The temperature error is therefore due entirely to the difficulty of determining the relation between change of length and temperature, on account of uncertainty as to whether the thermo-couples give the true temperature of the wire. It is probable that this error is not more than 5° C. at 150° C., 10° C. at 350° C., and 20° C. at 475° C. and I am of the opinion that it is considerably less. If we were to con-

[1] *Loc. cit.*

[2] *Loc. cit.*

[3] Annal. d. Phys., *141*, 481, 1871.

[4] Acta. Soc. Scien. Fennicae, *35*, 1908.

sider the curves as moved, first, with a motion up and down a distance corresponding to one per cent. of the modulus and, second, with a motion to the right and left corresponding to twenty degrees variation in temperature, the true position of the curve would fall well within the area covered.

As explained in a previous section of this paper there is a variation of temperature along the wire amounting to 40° C. at 350° C. Consequently at any given average temperature the value of the modulus is the average of the moduli of elements of length some of which are at temperatures lower and some at temperatures higher than the average. In the case of current heating the variation along the wire is less but a variation in the cross section is introduced. It is assumed that the change of length averages the variation in temperature of the different cylindrical layers as well as the variation along the length. It should be noted that the value of the modulus is a similar average of the moduli of the different layers as well as the different elements of length. Except for the fact that sudden changes in the modulus, or in the slope of the curve, peculiar to some definite temperature would be concealed by this averaging these factors are negligible in comparison with the errors already discussed.

Attention has been called to the effect of magnetization on the modulus and mention is now made of magnetostriction,[1] the cooling of the wire on application of the weight,[2] and the apparent increase in length of the wire accompanying the straightening of the catenary only to call attention to the extreme smallness of these effects and the fact that they may be entirely neglected in this investigation.

Summary.

The results of the tests that have been made upon this sample of mild steel wire may be summarized as follows:

1. The sample showed very erratic changes in elasticity when first heated.

2. By continued heating and stretching it was brought to a cyclic condition, or steady elastic state, in which Young's modulus becomes a function of temperature. This was secured first for a temperature range of 20° C. to 300° C., and later for a range extending to 475° C.

3. Continued heating and stretching gradually increased the modulus, the effect being probably caused largely by the stretching.

4. Except for the small gradual increase above mentioned, the modulus

[1] Honda and Terada, *loc. cit.* Honda, Shimizu, and Kusakabe, *loc. cit.* Dorsey, Phys. Rev., O. S., Vol. 30, 698, 1910. Brackett, Phys. Rev., O. S., Vol. 5, 257, 1897. Rhoads, Phil. Mag., Vol. 2, 463, 1901, and others.

[2] Joule, Proc. Roy. Soc., Vol. 8, 355, 1857; Phil. Trans., Vol. 149, 91, 1859; Scientific Papers, Vol. I, pp. 405, 413.

of the wire was independent of history, the thermal route by which any temperature was reached having no apparent influence upon the value of the modulus.

5. Heating by an electric current has no effect other than that caused by the accompanying temperature. This is in direct contradiction to the work of Walker. The very small changes in the modulus caused by magnetization reported by other investigators would not have been detected by this apparatus.

6. The Young's modulus of the wire decreased with increase of temperature at an increasing rate. The modulus decreased slowly and almost uniformly up to a temperature of about 300° C. Then the rate of decrease became more and more rapid, the plotted curve becoming very steep at 475° C. The following table is compiled from the various results.

TABLE II.

Change of Young's Modulus of a Mild Steel Wire with Increase of Temperature.

Temperature.	Modulus $\times 10^{-11}$ Dynes per Cm.^{2}	$\frac{dE}{dT} \times 10^{-11}$	Total Decrease, Per Cent. of E at 20° C.	Temperature.	Modulus $\times 10^{-11}$ Dynes per Cm.^{2}	$\frac{dE}{dT} \times 10^{-11}$	Total Decrease, Per Cent. of E at 20° C.
20° C.	19.3	.0035	0.0	300° C.	18.0	.0074	6.7
50	19.2	.0036	0.5	350	17.5	.0106	9.3
100	19.0	.0038	1.5	400	16.8	.0205	11.9
150	18.8	.0041	2.6	425	16.1	.0310	16.5
200	18.6	.0047	3.6	450	15.1	.0470	21.7
250	18.3	.0058	5.2	475	13.8	.0860	28.5

7. The values of the modulus are accurate to one per cent. for temperatures between 20° C. and 350° C. At 475° C. the error may amount to three or four per cent. Below 350° C. the temperature error is negligible in comparison to the error in the modulus. At 350° C. it may amount to 10° C. At higher temperatures the error is greater and of more importance on account of the more rapid decrease of the modulus. At 475° C. it is possible that the error may be as much as 20° C.

8. The above results while applying to but one sample of wire, when considered in the light of other investigations, appear to be characteristic and I should expect to find very similar effects with other samples of iron and steel.

In conclusion I wish to acknowledge indebtedness to the staff of the physical laboratory of the State University of Iowa for their interest in the work and especially to Professor G. W. Stewart for suggesting the problem.

PHYSICAL LABORATORY,
STATE UNIVERSITY OF IOWA,
JULY, 1914.

[Reprinted from the PHYSICAL REVIEW, N.S., Vol. V, No. 5, May, 1915.]

THE NATURE OF ELECTRIC CONDUCTION AS REQUIRED TO EXPLAIN THE RECOVERY OF RESISTANCE OF METALLIC SELENIUM FOLLOWING ILLUMINATION.

BY F. C. BROWN.

DIFFERENT theories have been proposed to explain the transportation of electricity in solids, but metallic conductors generally have not offered satisfactory evidence for a final decision between the theories. The illuminating experiments on thermionics by Richardson and his students and also the consistent agreement of theory and experiment involving the ratio of the electrical to the thermal conductivity of metals have made various modifications of the dynamical equilibrium theory very popular. The seemingly most contradictory evidence to this theory is the low value of the specific heat for all the metals. However, Sir J. J. Thomson[1] has shown that the calculated and experimental ratios of the electrical to the thermal conductivity may agree equally well by a theory that presumes the electrons to be tied up with the atom in the form of a doublet. In this case electric conduction would be merely the transportation of electron from doublet to doublet much in the same way as the Grotthus chains in the old theory of electrolysis.

I shall in this paper present information which leads away from the first mentioned view above and which forms the basis for a theory bearing more or less resemblance to the above mentioned second theory of Thomson. For the reason that the conducting power of metallic selenium crystals may be varied by so many physical conditions, we have the obvious opportunity of making very definite choice as to what form of the electron theory is satisfactory. Having obtained a consistent theory for selenium, we can then decide if reasonable modifications of the theory will explain conduction in the other elements, where there are less favorable means of attack.

The investigations described in this and succeeding papers will be somewhat along the line of attack made by J. W. Nicholson.[2] An effort will be made to choose experiments that enable decision to be made with

[1] Book on the Corpuscular Theory of Matter, 1907.

[2] PHYS. REV., N. S., Vol. 3, p. 1, 1914. See also article by Merritt, PHYS. REV., 25, p. 505, 1907, which involves some of the elements of the ideas in this paper.

a minimum number of assumptions. The author is not aware of any direct or indirect measurements having been made which consider any particular value for the rate of recombination of electrons with their positive residues, except in the conduction of electricity through a gas under the influence of an ionizing agent. The subject will be opened by a consideration of the rate of recombination of the electrons and some of the conditions that influence this rate.

THEORETICAL CONSIDERATIONS.

It will be assumed that the specific conductivity varies as the number of electrons taking part in the conduction at any instant. The electrons do not exist free in the sense of the kinetic theory of gases. Under illumination they are rendered unstable or free but on the average they recombine with the atomic structures very rapidly. Experiment shows that following intense illumination thirty per cent. of the extra electrons return to their fixed positions in .02 second. A second assumption that will be made is that the recombination of the freed electrons will take place according to the same law governing the recombination of ions in gases. Then the rate of recombination of electrons will be expressed as,

$$\frac{dN}{dt} = -\alpha N^2, \tag{1}$$

where N is the average number of electrons in the free state at any instant and α is the coefficient of recombination. However, this equation holds only for a uniform distribution of the electrons. Such a distribution would exist when the selenium reaches the equilibrium condition in the light, or for the first small interval of time, Δt, after the illumination is shut off. The theory will presuppose that the coefficient of recombination is not altered by the condition of light or dark, and the experiments will verify this presumption.

Since the conductivity is proportional to the number of free electrons, we may write,

$$i = k_1 \cdot N \tag{2}$$

from which it follows that

$$\frac{di}{dt} = -\frac{\alpha i^2}{k_1}, \tag{3}$$

or

$$\frac{di}{dt} = -\alpha' i^2,$$

where α' is merely a new constant.

When the selenium is in equilibrium in the light the rate of recombination of the electrons is exactly equal to the rate of production of

electrons by the light in addition to the natural production in the dark. This may be represented as

$$\frac{dN}{dt} = M + q,$$

or

$$\frac{di}{dt} = k_1(M + q), \tag{4}$$

where M is the former rate and q is the latter or dark rate. Combining equations (3) and (4) we obtain, for the equilibrium value of the current,

$$i_e = k_1\sqrt{\frac{M + q}{\alpha}}. \tag{5}$$

Now we could check the theory by the application of observed data to satisfy equation (5), but the rate of production of electrons by light, M, is dependent upon the light intensity and therefore it will be more convenient to test the adequacy of the equation in the next succeeding paper, together with the law governing the rate of production with varying light intensities. It will be the purpose of this paper to verify the fundamental relation of rate of recombination to the number of free electrons, as expressed in equations (1) and (3). It was found that equation (3) could be verified when expressed in the approximate form

$$\frac{\Delta i}{\Delta t} = -\alpha' i^2, \tag{3}$$

when Δt was kept a very small interval of time and constant. Ordinarilly we should expect to check this equation by the application of its integrated form,

$$\left(\frac{1}{i} - \frac{1}{i_0} = \alpha t\right)$$

to a complete recovery curve extending over a considerable length of time. However, I have found that this can not be carried out satisfactorily because as soon as a large percentage of the electrons have recombined, a non-uniform distribution of the electrons and uncombined atoms exists, such that the coefficient of recombination is diminished. But it is not essential to the argument of this paper to either prove or disprove this statement. A slow diffusion of the electrons, and changing crystalline structure, no doubt are complexities to be taken into account in explaining a complete recovery curve.

That equation (3) is tenable, together with the underlying assumptions mentioned, may be ascertained from some observations taken with masses of crystals some years back. In my paper on the " Recovery of the Giltay Selenium Cell and the Nature of Light Action in Selenium "[1] on p. 415

[1] Phys. Rev., Vol. 33, p. 403, 1911.

is just such data as desired. The conducting component, B, has the same significance as i or N in this paper. The recovery during the mean interval of .05 second was measured after the selenium had begun to recover from illumination of varying intensities. The varying intensity produced the varying conductivity noted in the following table.

Conductivity in Dark. i_0	Conductivity in Light. i	Change of Conductivity, Ohms^{-1}. $\Delta i = k \cdot N$	$\frac{\Delta i}{i}$	$\frac{\Delta i}{i^2}$	Coefficient of Recombination. $\alpha' = \frac{\Delta i}{i^2 t}$
1.47×10^{-6}	6.29×10^{-6}	0.016×10^{-5}	2.5×10^{-2}	4.0×10^{3}	80×10^{3}
	$13.5 \times$ "	$0.11 \times$ "	$8.2 \times$ "	$5.9 \times$ "	$118 \times$ "
	$60.2 \times$ "	$2.79 \times$ "	$46.5 \times$ "	$7.6 \times$ "	$152 \times$ "
	$35.4 \times$ "	$1.05 \times$ "	$29.6 \times$ "	$7.9 \times$ "	$158 \times$ "
	$50.8 \times$ "	$2.04 \times$ "	$40.2 \times$ "	$7.4 \times$ "	$159 \times$ "
	$16.1 \times$ "	$0.18 \times$ "	$11.2 \times$ "	$6.9 \times$ "	$138 \times$ "

It may be observed, where the change of conductivity varies over the extreme range of from .016 to 2.79 (*i. e.*, by a factor of 174) that $\Delta i/i^2$ is approximately a constant. The values of α' as recorded in the last column are slightly in error because no allowance is made for the liberation of electrons taking place in the dark simultaneously with their recombination. If no new postulates are involved this correction should be of magnitude, $(i_0^2 \cdot \alpha' \cdot \Delta t)$, when added to Δi for the calculation of α'. Since this correction involves an error of less than ten per cent. in any value above, we will not complicate the argument of this paper by the application of this correction or considerations of the adequacy of the correction. Since the range of application of the data is so great, we may regard the constancy of the coefficient of recombination as satisfactory evidence that the electrons recombine with the atomic structures in accordance with the conception involved in equation (1), when the conditions are as specified. Since writing this paper I have also verified this fundamental conception by experiments on the recovery of single isolated crystals of selenium. This agreement of behavior of crystals and crystal aggregates is quite consistent with the other unique properties existing in the crystals, such as the likeness of the wave-length,—sensibility curves.[1]

It may be noted that a constant coefficient of recombination involves the idea that the number of electrons freed in dark recombine at a more rapid rate, when the selenium is illuminated or just following illumination. This follows because there are more positive residues and consequently more chances for recombination.

It should also be noted that Plimpton[2] has observed that ions in gases

[1] PHYS. REV., N. S., Vol. 4, p. 507, 1914; Vol. 5, p. 65, 1915.

[2] Am. Journ. of Sc., Vol. 35, p. 39, 1913.

also recombine according to the same law involved in equation (1), only in case there is a uniform distribution of ions. However, this agreement of the law of recombination of electrons in selenium with the recombination of ions in gases does not further lead us to suspect that the electric current may be transported by the same method in both selenium and in gases. The current in selenium with a given low potential is infinitely larger than any ionization current in gases, except in spark discharge. Further differences and likenesses of the method of transportation of the current will be brought out in later developments of the theory.

The Inadequacy of Ohm's Law.

A fundamental property in metallic selenium exists in the inadequacy of Ohm's Law to explain the variation of the current with the applied voltage. In all cases the current increases more rapidly than the proportional increase of the voltage. It is therefore pertinent to inquire what conditions are responsible for this unique property. According to the theory involved in equation (5) an increased potential can vary the specific conductivity only in two ways, by varying the rate of production of the electrons or by varying the rate of recombination of the electrons.

We will suppose that the increase of current necessary to satisfy Ohm's Law, when the voltage is increased, arises from increased velocity of drift of the electrons, and that the slight excess current arises from additional electrons in the conducting state. This increased number might come either from a magnified rate of production or from a diminished rate of recombination.

The increased rate of production might be expected because of bombardment of semi-fixed electrons by the faster moving ones or the greater electric intensity might be considered as lowering the degree of stability of all the electrons of a certain class in the atomic structure. Consistent with either of these views, it would be reasonable to expect a diminished rate of recombination as the voltage is increased. An increased velocity of drift would lessen somewhat the chance of an atom to capture an electron and also a lower stability of the atom would indicate a smaller attractive force for the electron.

A measurement of the recovery during a short interval following the extinction of the illumination should determine whether the coefficient of recombination varies with the applied potentials in such a manner as to explain the inadequacy of Ohm's Law.

The change of conductivity during short periods of recovery was measured by the Wheatstone's bridge and pendulum method.[1] The light

[1] Phys. Rev., Vol. 33, p. 54, 1911.

was cut off the selenium at the desired time by an aluminum shutter attached to the timing pendulum. A single lamellar crystal of metallic selenium of the fifth system was used for this investigation. The intensity of illumination from which the crystal recovered was kept constant. Also the pressure on the crystal was fixed at such a value that the resistance was 1,349,000 ohms with 1.45 volts. The change of conductivity was measured for the first 0.05 second interval of recovery, both when 1.45 volts was the difference of potential across the crystal and when there was 20 volts.

The change of conductivity during recovery of 0.05 seconds is shown in the following table. For convenience 0.05 second is here considered as the unit of time.

	With 20 Volts.	With 1.45 Volts.
Resistance in dark	1,192,000	1,349,000
Resistance in unvarying light	356,000	392,000
Conductivity in light	$i = 28.1 \times 10^{-7}$	$i = 25.5 \times 10^{-7}$
Recovery after 0.05 second	$i = 15.65 \times 10^{-7}$	$i = 14.1 \times 10^{-7}$
Change of conductivity	$\Delta i = 12.5 \times 10^{-7}$	11.4×10^{-7}
$\alpha' = \frac{\Delta i}{\Delta t \cdot i^2}$	1.58×10^5	1.75×10^5
$\frac{\alpha'}{i} = \frac{\Delta i}{i} = \frac{\Delta N}{N}$	0.447	0.447

It may be observed that the coefficient of recombination is not constant. In fact a brief consideration of the data reveals that this coefficient varies directly as the specific resistance of the crystal, when the variation of the specific resistance accrues from an altered potential difference between the crystal electrodes. This conclusion is verified by the constant ratio of α'/i, as recorded in the last row above.

If the constant, α', were the sole quantity that changed its value with varying potentials across the crystal, then we might expect in accordance with equation (5) that the equilibrium light sensitiveness of the crystal would vary inversely as the square root of α. However, the data for the same crystal that is given in the following table shows that the equilibrium light sensitiveness is almost proportional to the conductivity in the light. Since (α) varies inversely as the same conductivity in the light, and since the conductivity is influenced only by the square root of the recombination constant, it follows that the light sensitiveness should, according to equation (5), vary directly by some function of $M + q$, and further that this function should have the same value as the function relating (α) to the conductivity. This idea is a little beyond the province of this paper and will need to be verified further.

Potential across the crystal in volts	1.5	6.0	36	60
Conductivity in dark	5.9×10^{-7}	6.0×10^{-7}	6.85×10^{-7}	7.5×10^{-7}
Conductivity in light	20×10^{-7}	20.4×10^{-7}	23.3×10^{-7}	24×10^{-7}
Equilibrium light sensitiveness	14.1×10^{-7}	14.4×10^{-7}	16.5×10^{-7}	$16.5 - 10^{-7}$
$\frac{\text{Light sensitiveness}}{\text{Conductivity in light}}$	.705	.70	.70	.69

The Pressure Effect.

The increase of the specific conductivity of selenium, by pressure, whether in isolated crystals or crystal aggregates is another unique principle that our electron theory should explain.[1] The specific conductivity may vary a hundred fold by increasing the pressure. According to the postulates in the earlier part of the paper, this increase of conductivity must arise either from an increase in the number of electrons capable of taking part in the conduction or in a decrease in the ratio of recombination of the electrons with the positive residues. From a consideration of the increase of the absolute light sensitiveness with increased pressure I have already concluded[2] that it would be unreasonable to expect the light-sensitiveness to increase in proportion to the conductivity in the dark, if the conductivity must vary alone with the number of electrons liberated.

Since there is no reason to expect any large changes of conductivity resulting from variations in the free path of the electron, it seemed very plausible that pressure might alter the rate of recombination of the electrons.

In the experimentation, the selenium crystal was placed between brass electrodes and the variable pressure desired was controlled by a screw adjustment. At each pressure the equilibrium conductivity was measured both with the crystal in the dark and with constant illumination. The recovery was measured during the first mean period of .02 second after the extinction of the illumination as elsewhere described. A constant difference of potential of 13 volts was kept between the crystal electrodes. The equilibrium conductivity in the light was attained in less than a second after illumination.

The following table shows the value of the coefficient of recombination for various pressures on the crystal such that the conductivity varied from 1.79×10^{-7} to 23.8×10^{-7} in the dark.

Since the pressure effect is of such large magnitude as noted, we may conclude (from the constant value of $\alpha' \cdot i$) with some certainty that the

[1] Brown and Stebbins, PHYS. REV., Vol. 26, p. 273, 1908; PHYS. REV., N. S., Vol. 4, p. 85. 1914.

[2] *Loc. cit.*, article above.

Pressure.	Conductivity in Dark Ohms^{-1}.	Conductivity in Light Ohms^{-1}.	Recovery in .02 Sec. Change of Conductivity. $i = K \cdot N$	$\alpha' = \frac{\Delta i}{i^2 \cdot \Delta t}$	$\alpha' \cdot i$
P_1	1.79×10^{-7}	5.18×10^{-7}	1.84×10^{-7}	3.5×10^{7}	17.9
P_2	$6.50 \times$ "	$15.4 \times$ "	$4.12 \times$ "	$1.35 \times$ "	17.1
P_3	$12.2 \times$ "	$30.3 \times$ "	$9.02 \times$ "	0.445	14.2
P_4	$18.9 \times$ "	$43.9 \times$ "	$14.36 \times$ "	0.300	15.0
P_5	$23.8 \times$ "	$55.5 \times$ "	$15.5 \times$ "	0.26	14.0

coefficient of recombination varies inversely as the initial conductivity in the light, that results from the pressure.

Now it may be observed in the same table of data that the light sensitiveness at the different pressures is almost proportional to the conductivity in the dark accompanying each pressure. If, in equation (5), the value of q were small compared with M, it would be necessary for M (the rate of production by a fixed illumination) to vary directly with the initial conductivity at any pressure in order that this increase of light sensitiveness might be proportional to the dark conductivity.

We have, therefore, proved that the coefficient of recombination varies inversely as the conductivity ensuing from the pressure, and the evidence just stated inclines very much toward the view that the rate of production by light varies directly as the same conductivity resulting from the pressure effect.

General Considerations.

The attempt has been made to build up a simple electron theory to correlate the most fundamental photo-electro-mechanical properties in metallic selenium. The experiments have been carefully selected to verify the basic conceptions with a minimum number of postulates, by avoiding all questions of absorption, reflection and time rate of change of conductivity.

The effort has been successful in achieving a simple consistency of results. It was first shown that the electrons do recombine very rapidly with what is supposed to be the atomic structures, and the recombination takes place according to the same law governing the recombination of ions in gases. But we can not conclude from this that the current is transported in the same way in selenium as an ionization current in gases.

The basis of the theory receives further support in that a common explanation accounts both for the variation from Ohm's law and for the large changes of conductivity accompanying pressure changes. In each case the coefficient of recombination of the electrons is found to vary inversely as the variable conductivity imposed by the pressure change or the difference of potential. And likewise, there is common evidence that the rate of production of conducting electrons by a fixed illumination is

directly proportional to the variable conductivity imposed by pressure or voltage.

The temporary liberation of electrons by light bears certain resemblance to the original suggestion of Pfund[1] in which he compared light-action in selenium to an internal photo-electrical effect. This idea also is consistent with the experiments of Dr. L. P. Sieg and the author[2] where we have found all isolated crystals of selenium to have a maximum sensibility in the ultra-violet region of the spectrum.

But the experiments indicate a distinctly new idea as involved in a coefficient of recombination of the electrons that varies with the physical conditions surrounding the crystal, and also a varying rate of production of electrons with a fixed light intensity. This varying rate of freeing of electrons is governed by the same law apparently as that governing the variation of the recombination constant, except that the two relations are in inverse direction.

The form of the electron theory as here presented offers a satisfactory explanation of the electro-dynamical, light-electrical and electrical relations recently published.[3] The action due to pressure is not transmitted beyond the region of stress. The pressure merely lowers the stability of the selenium such that a given light intensity may liberate a greater number of electrons where this stress exists and such that the electrons recombine less rapidly in the same region.

Consistent with this interpretation, the light action transmitted to a distance is increased if the pressure is applied to the portion of the crystal where the conduction takes place, but the transmitted effect is not increased by pressure applied only at the place of illumination. The transmitted light action is of the nature of a crystal disturbance, which lowers the stability of the electrons everywhere in the confines of the crystal.

Since the analysis has shown the voltage effect to be identical with the pressure effect so far as the rate of recombination of electrons or their liberation is concerned, it is to be expected that the voltage effect could not be transmitted throughout the crystal in the way that light action is transmitted. This, in fact, is the result found in the previous work.[4]

Naturally the theory suggests many other lines of investigation in order to obtain more detailed information as to the nature of electric conduction in selenium. Since we have already considered such a wide range of experiments and have obtained such satisfactory agreement, the theory is one of unusual promise.

[1] PHYS. REV., Vol. 28, p. 234, 1909.
[2] PHYS. REV., Vol. 4, p. 48, and p. 507, 1914.
[3] PHYS. REV., N. S., 1914.
[4] *Loc. cit.*

[Reprinted from the PHYSICAL REVIEW, N.S., Vol. V, No. 5, May, 1915.]

SOME EXPERIMENTS ON THE NATURE OF TRANSMITTED LIGHT-ACTION IN CRYSTALS OF METALLIC SELENIUM.

BY F. C. BROWN.

RECENTLY we showed[1] that light falling on one part of a crystal of selenium would produce a change of conductivity throughout the crystal. In the acicular crystals this effect was observed practically undiminished in amount as far as 10 mm. away from the point of illumination. This effect was denoted by the authors as a new property in matter. The work described in this paper consists essentially of two investigations designed to give information concerning the nature of this light-action. The first was an experiment to determine the velocity of transmission of the light effect along the crystal, and the second was a study of certain interrelated phenomena between the pressure effect[2] and the transmitted light action. The one showed the action to be transmitted much too rapidly for a temperature effect. The other definitely proved that the increase of conductivity at a distance could not arise from transmitted free electrons. Incidently, the results call forth a new view as to the nature of electrical conduction as exhibited in crystals of metallic selenium.

THE RATE OF TRANSMISSION OF LIGHT-ACTION ALONG THE CRYSTALS.

To obtain information as to the rate of transmission of this new effect the method used was to determine the resistance after short intervals of time following illumination at a distant point. A lamellar crystal of the fifth system, of size about 4 × .6 × .3 mm., with striations perpendicular to the length of the crystal was mounted with opposite ends between separate sets of electrodes as shown conventionally in Fig. 1. Under crossed nicols the crystal would show parallel extinction. A constant source of illumination was obtained by focussing a Nernst glower on the crystal.

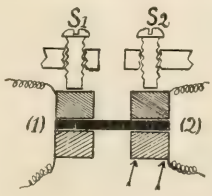

Fig. 1.

[1] Brown and Sieg, Phil. Mag., Ser. VI., Vol. 28, p. 497, 1914; Brown, PHYS. REV., N. S., Vol. IV., p. 85, 1914.

[2] For a description of this effect see paper by author in PHYS. REV., Ser. 2, Vol. 14, p. 85, 1914.

The procedure was to connect the resistance between electrodes at end (2) in one arm of a Wheatstone's bridge circuit. The resistance of this part of the crystal was measured in the dark and when it was illuminated, and then the same part was measured again when the opposite end (1) was in the dark and then in equilibrium under the same constant illumination as was previously on the conducting end. Next the above procedure was repeated several times, with the modification necessary to measure the change of resistance during the first 0.4 second of illumination. The object was to determine what part of the total change of resistance was transmitted two millimeters along the crystal in this short interval. Of course, even if the light falls directly on the part of the crystal whose resistance change is under determination all the effect does not take place at once.[1] Thus the opposite ends of the crystal were illuminated alternately in order to find the relative lag in the transmitted effect.

The method of measuring the change of resistance in these small intervals was that described by Brown and Clark.[2] A shutter was attached to a ballistic pendulum, which automatically put a galvanometer in circuit for a short interval at any desired time after the shutter moved out of the path of the beam of light directed on the crystal. Thus in Table I. the change of resistance was recorded as divisions throw of the galvanometer, which was afterward reduced to ohms.

In the first series of observations, where the conductivity of end (1) of the crystal was measured there is a rather wide variation of the readings, but this was brought about by a deliberate variation of the lighting arrangement, extra screens to cut off stray light, etc., being used. The purpose was to make observations under corresponding conditions when each end was illuminated.

The result of these experiments can be stated as follows:

Both the direct and the transmitted actions are very rapid, more than 50 per cent. of the equilibrium change taking place in 0.4 second. In searching for an explanation of the fact that the percentage change of conductivity in 0.4 second was quite different depending on which end of the crystal was tested, facts were discovered which indicate that the ratio of the area illuminated to the cross sectional area conducting is a factor in the rate of change in short intervals of time. Whatever may be the outcome of a study of this relation, it is not believed that the accompanying results will be at all vitiated. Second, that the total amount of the transmitted action is of the same order of magnitude as the direct action and yet distinctly less. Third, the difference between

[1] See PHYS. REV., Vol. 33, p. 403.

[2] PHYS. REV., Vol. 33, p. 53, 1911.

TABLE I.

Change of Resistance of Crystal at End (1).

	Illuminated at Electrode (2), Ohms.	Illuminated at (1), Ohms.
Resistance in dark	880,000	880,000
Resistance in equilibrium with light	450,000	370,000
Change of resistance at equilibrium	430,000	510,000
Change of resistance in 0.4 sec	7.0	9.0
	7.5	9.0
	7.0	11.5
	9.0	12.0
	9.0	12.5
	8.0	12.0
	7.5	12.0
	9.0	15.0
	11.0	15.0
	11.0	18.0
Mean in div.	8.6	12.9
Mean ohms	183,000	234,000
Ratio, $\frac{\text{change in 0.4 sec.}}{\text{equilibrium change}}$	.426	.459

Change of Resistance Measured at End (2).

	Arranged to Illuminate end (1).	To Illuminate (2).
Resistance in dark	510,000	510,000
Resistance in equilibrium in light	340,000	260,000
Change of resistance at equilibrium	170,000	250,000
Change of resistance in 0.4 sec	11.0	18.0
	10.5	17.5
	11.0	16.5
	11.0	16.5
	10.0	14.5
Mean in div.	10.7	16.6
Mean in ohms	136,000	215,000
Ratio, $\frac{\text{change in 0.4 sec.}}{\text{equilibrium change}}$	.80	.86
Change of resistance in 0.2 sec	10. div.	16.
	10.5	15.5
Mean in ohms	133,000	201,000
Ratio, $\frac{\text{change of resistance in 0.2 sec.}}{\text{equilibrium change}}$	.78	.80
Change of resistance in 0.1 sec	7.0 div.	13.5
	7.5	12.5
	7.5	12.5
Mean	7.3	12.8
In ohms	95,000	162,000
Ratio, $\frac{\text{change in 0.1 sec.}}{\text{equilibrium change}}$	.56	.65

the fractional parts of the total change taking place in 0.4 second for the direct and the transmitted effects is very small. The proportional change in 0.2 second is also observed to be the same within the limits of accuracy of the measurement. Even for 0.1 second exposure there is almost as great a fraction of the effect transmitted to the opposite end of the crystal as at the illuminated end.

The conclusion is fairly safe that practically all of the transmitted action by light may travel a distance of 2 mm. in less than 0.1 second. How much faster than 2 centimeters per second it may travel, I was not prepared to determine. At any rate the effect travels so fast that we are warranted in saying that it can not be transmission of a temperature change along the crystal. This conclusion is quite in agreement with recent experiments by Sieg and Brown,[1] in which it was shown that for equal quantities of energy in different parts of the spectrum falling on the crystal, a maximum transmitted effect occured in the visible spectrum, not far from the position where the maximum occurred for direct action of the light. Likewise, if the transmission is too rapid to be a transmitted temperature disturbance, it must, according to the electron theory, be too rapid to be merely an equalization of electronic pressures throughout the crystal.

The Action at a Distance is Propagated Mechanically.

The fundamental fact is that light falls on a crystal of selenium at one spot and produces a change of conductivity at any other part of the crystal. It is inconceivable that the light itself could, on entering a crystal, diffuse almost without absorption to the most distant part of a crystal, and yet such may be the case. Therefore the nature of this transmitted effect was investigated along other lines. One view would suppose the light by virtue of its electromagnetic properties to be able to directly tear the electrons free from the atomic structure. In order that there might be almost undiminished action at a distance, either these electrons must disperse to all parts of the crystal structure or at the place where the light falls there must be an increased concentration of electrons which would quickly be felt throughout the crystal, the same as an increased quantity of gas in one part of a tank system would be felt everywhere in the enclosure. The velocity of such a disturbance would be largely a function of the elastic properties of the electrons in confined space. Another view is that the light acts upon a certain mechanism which produces automatically a certain instability throughout the crystal structure. This instability manifests itself by a greater electrical con-

[1] PHYS. REV., N. S., vol. 4, p. 507, 1914.

ductivity, which means either an increased number of free electrons or a greater instability of the fixed electrons. The first view involves the direct carrying over of the action without the aid of the crystal structure as such while the second view involves something analogous to an elastic medium propagation.

The merits of the above views were investigated largely with the use of the arrangement shown in Fig. 1. In my previous paper it was pointed out that the resistance of a crystal varies with the mechanical pressure under which it exists and also that the resistance varies with the potential differences producing the current. With this apparatus were studied the interaction of various agents, viz. light, pressure and electrical potentials, that alter the resistance of the crystal.

It was found, no matter how much the resistance might change at end (1) as a result of large differences of potential there, that the resistance at the opposite end (2) did not vary. Similarly pressure on end (1) of the crystal by the screw S_1 changed the resistance at (1) by a factor of ten but the resistance at the opposite end was thereby changed only by a zero or negligible amount. Thus we have the clear cut result that *light-action is transmitted along the crystal, but the pressure effects and the electrical potential effect*, as I have designated them, *are not transmitted.*

A most important part of the experiment was in the superposition of the pressure and the light effects. In this experiment only end (1) of the crystal was illuminated in all the observations. The conductivity was measured at both ends simultaneously, both when end (1) was in the dark and when it was illuminated. The observations are shown in Table II. The pressures were deduced from the conductivity values according to the relation found in an earlier paper.[1] The illumination was practically constant throughout. A brief study of the table will verify the following generalization: *the increase of pressure increases the light sensibility* (*i. e., the change of conductivity due to constant illumination*) *only when the pressure is applied to the part of the crystal where the conductivity is being measured.*

From the results stated we are warranted in making the following deductions: If electrical conduction in these crystals is due to *free electrons that exist in equilibrium according to the Maxwell-Boltzman law, it can not be possible that the mechanical pressure in increasing the conductivity increases the number of free electrons.* This follows because the pressure effect is not transmitted from one part of the crystal to another and because the light-sensitiveness with varying pressure remains constant everywhere except at the points where the pressure is applied. There might, of course,

[1] PHYS. REV., Ser. 2, Vol. 4, p. 85, 1914.

TABLE II.

Conductivity of Lamellar Crystal, $\times 10^7$.

	Between Electrodes at (1).	Between Electrodes at (2).
In dark	4.0, 4.0, 4.0	4.8, 4.5, 5.0
End (1) illuminated.	7,8, 8.0, 8.0	6.9, 7.4, 7.7
Increase of conductivity	3.8, 4.0, 4.0 mean 3.9 div.	2.1, 2.9, 2.7 mean 2.6 div.
Pressure on (1) 3, kgm./cm². on (2) 2 kgm.		
Pressure increased by S_1.		
In dark	8.0, 8.8, 8.8	5.2, 5.3, 5.3
End (1) illuminated.	16.0, 16.6, 16.4	7.8, 7.8, 7.8
Increase	8.0, 7.8, 7.6 mean 7.8	2.6, 2.5, 2.5 mean 2.6
Pressure on (1) 6, on (2) 2 kgm./cm².		
Pressure increased by S_1.		
In dark	28.0, 31.6, 31.6	5.3, 5.3, 5.4
End (1) illuminated.	52.8, 52.6, 52.2	7.3, 7.5, 7.2
Increase	24.8, 21.0, 20.6 mean 22.1	2.0, 2.2, 1.8 mean 2.0
Pressure on (1) 18 kgm./cm²., on (2) kgm./cm².		
Pressure increased by S_2.		
In dark	20.0, 24.0, 26.0, 25.6	20.0, 20.0, 25.0, 25.0
End (1) illuminated.	40.0, 42.0, 48.0, 47.0	23.2, 30.3, 33.0, 34.5
Increase	20.0, 18.0, 22.0, 21.0 mean 20.2	3.2, 7.3, 8.0, 9.5 mean 7.0
Pressure on (1) 18 kgm./cm²., on (2) 6 kgm./cm².		
Pressure increased by S_2.		
In dark	25.4, 26.0, 26.4	68.9, 71.4, 74.0
End (1) illuminated.	48.8, 48.6, 47.0	81.3, 86.9, 88.5
Increase	23.4, 22.0, 20.6 mean 22.0	12.4, 15.5, 14.5 mean 14.5
Pressure on (1) 18 kgm./cm²., on (2) 13 kgm./cm².		

be a transmission of the pressure effect of secondary magnitude and importance which would not be detected except in more highly refined work. Then at least that part of the conduction that is brought about by increased pressures can not result from an increase in number of dynamically free electrons, and likewise that part of the increased conductivity that comes from a constant illumination as a result of increased pressure can not arise from free electrons at constant pressure everywhere within the crystal. Of course this argument requires that the increased pressure reacts against the fixed crystal structure and not against the free electrons. Now if the increased conductivity resulting from pressure on the crystal is not due to free electrons, it is difficult to justify conduc-

tion by free electrons at atmospheric pressure. The conclusion then seems unavoidable *that electrical conduction in crystals of metallic selenium can not be due to the traditional free electron.*

This conclusion need not be inconsistent with the result of Richardson and Brown[1] that the electrons inside a metal are free in the sense of the kinetic theory of gases, for our result was based upon work with highly conducting metals. Perhaps conduction in all non metals is like that in selenium crystals and dissimilar to that in the good conductors. It might be urged that the conductivity of selenium crystals is a function both of the number of free electrons and of a resisting medium through which they must pass. But this particular motion is inconsistent with the rapid transmission of the light-action along the crystal as was found.

The most satisfying unification of the experiments related that I have been able to conceive rests upon the hypothesis of conduction by electrons in semi-stable equilibrium. Scattered throughout the crystal structure are centers, perhaps atomic center, in which are associated charges of electricity in almost unstable equilibrium. Electrons free to move about in the structure as gas molecules move in enclosure do not exist. True these electrons are fixed in number and in position in the crystal structure, but the degree of their stability will vary with the agencies acting on the crystal. Electrical conduction consists essentially of a pulling out of these electrons from their moorings in the direction of the electrical stress. While out of position an electron might behave temporarily as a free electron in equilibrium with the heat and electrical forces about it. This process of conduction bears a little resemblance to the transfer of electricity in electrolytes.

The fact that Ohm's law does not hold for these crystals or metallic selenium generally is against the free electron hypothesis. The conductivity increases very greatly as the electrical forces in the line of conduction increase, until a saturation value of the conductivity is reached.

On this view increased pressure or tension on the selenium reduces the electrons to an average lower degree of stability. Thus a given fall of potential across the crystal will be able to dislocate a larger number of electrons from their fixed positions, or will be able to use them on an average a longer time before they recombine.

Similarly, light by some mechanism yet undiscovered lowers the degree of stability of the electrons throughout the crystal or further the mechanism controlled by light frees the most unstable electrons throughout the selenium. Thus as found if pressure is applied to any part of the crystal and any other part of the crystal is illuminated, that part of the

[1] Phil. Mag. (6), Vol. 16, p. 353, 1908, and Phil. Mag. (6), Vol. 18, p. 649, 1909.

crystal under pressure, and only that part, has its absolute light-sensitiveness increased. This merely means that at the place of great pressure the mechanism of light finds a greater number of electrons in such a low degree of stability that more of them can be kept in the free state.

It is still to be investigated how the light-action may be transmitted to a distance. It has occurred to the writer that it may be a change of crystalline structure, or an elastic vibration, or merely light diffusion.

[Reprinted from the PHYSICAL REVIEW, N.S., Vol. VI, No. 3, September, 1915.]

AN ATTEMPT TO DETECT A CHANGE IN THE HEAT CONDUCTIVITY OF A SELENIUM CRYSTAL WITH A CHANGE IN ILLUMINATION.

BY L. P. SIEG.

Introduction.—The fact that selenium changes its electrical conductivity so markedly with illumination has no doubt led many investigators to consider the possibility of a change in its heat conductivity with a change in the illumination. This has been of especial interest since the development of the electron theories for both kinds of conduction. However, careful search has revealed but one paper; that by Bellati and Lussana[1] bearing directly on the subject. These authors worked with a thin disc of crystallized light-sensitive selenium. The disc was painted with a thin coat of the red double iodide, Cu_2I_2, HgI_2. This substance turns chocolate brown at about 70° C. and so the color serves as an indicator of the extent on the disc where the above temperature has been attained. They heated a point of the disc with an electrically heated platinum wire. In the sunlight the increase in the thermal conductivity over that in the dark amounted, in one case cited, to about ten per cent. They noted further that the percentage change in the thermal conductivity agreed well with the percentage change in the electrical conductivity where exposure was made to the same light.

Some experiments of my own, performed on a selenium crystal are in serious disagreement with the above conclusions, and are here presented.

Theory of the Experiment.—It seems very questionable whether any simple electron theory is capable of explaining the light electric actions in selenium. Even assuming constant illumination one should be slow to adopt any of the standard electron formulas for heat and electric conduction. Lorentz,[2] Drude,[3] Livens,[4] Thompson,[5] Wilson,[6] and others have developed electron formulas for thermal and electrical conduction in metals. It will not be necessary to repeat these formulas here. It is

[1] Atti del R. Inst. Ven. (6), 5, 19, 1887. Abs. Beib. d. Ann., 11, 818, 1887.

[2] Theory of Electrons, pp. 67, 266.

[3] Ann. d. Phys., 1, 566, 1900; 3, 369, 1900.

[4] Phil. Mag., 29, pp. 173, 425, 1915.

[5] Phil. Mag., 14, 217, 1907.

[6] Phil. Mag., 20, 835, 1910.

to be noted, however, that although these formulas differ in the constants involved, they agree in the essential matter that the ratio of the electrical to the thermal conductivity is a function of the temperature, and is independent of the number of free electrons, their velocities, or their mean free paths. The classic experiments of Jaeger and Disselhorst[1] have given ample experimental verification of this independence. One might expect then that any agency that increases the electrical conduction of selenium, might be expected to increase in a measure the thermal conduction. To be sure we should not expect to find as large an increase for the thermal conductivity as for the electrical conductivity, for selenium is at best a very poor conductor of electricity, and so doubtless a large share of the heat conduction is by means of atoms and molecules, just as is true in the case of electric insulators. Nevertheless, if under the action of light there are more electrons *actually made free*, or if those free have a *greater mean velocity* or *path*, there should be at least a small increase in the thermal conductivity. The experimental results described below show, however, that if there is any such improvement in the heat conductivity it is smaller than the error of these measurements.

Apparatus and Method of Observation.—A modified form of the apparatus described many years ago by Christiansen[2] was used. A form of apparatus was required that would meet several conditions. It was decided to work with an isolated crystal of selenium.[3] It was necessary that the apparatus serve to obtain the heat conductivity and the electrical conductivity both in the dark and in the light, and further if possible that it should serve to get the variation of the above quantities with the temperature. A reference to the figure will make the following description clear. At the base is a brass cylinder through which passes a stream of cold water. This cylinder is about 50 mm. in diameter by 20 mm. high. On top of this are placed in succession the following layers. Outside, and in the form of washers we have, a copper plate, a glass plate, a copper plate, a glass plate, and a copper plate. Inside, and in the form of small discs we have the following five layers: a copper disc, a glass disc, a copper disc, a selenium crystal, and a copper disc. The total height of all five discs is 4.5 mm. The other necessary dimensions are indicated on the figure. The purpose of the outside washers was to form a guard ring having about the same temperature gradient as is to

[1] Preuss. Akad. Wiss. Ber. Sitzungsber., 38, 719, 1899, also Phys. Tech. Reich., Wiss. Abh. 3, 269, 1900.

[2] W. Ann., 14, 23, 1881.

[3] The crystal was kindly given to me by my friend Dr. F. C. Brown. The crystal was a close grown fern-like one made up of small, and very probably, hexagonal spines. For further description see PHYS. REV., Ser. 2, 4, 85, 1914.

be found in the central pile of discs. The various washers and discs were joined together with thin films of glycerin in order to improve the heat conductivity of the junctions. Above the whole is an electric heater. In practice this was separated from the top copper disc and washer by thin cover glass for purposes of electric insulation. Thermo-couples end in the central copper discs, the wires passing through the outer washers as shown, and insulated from them. The solid lines indicate copper wires and the dot and dash lines constantan wires. The

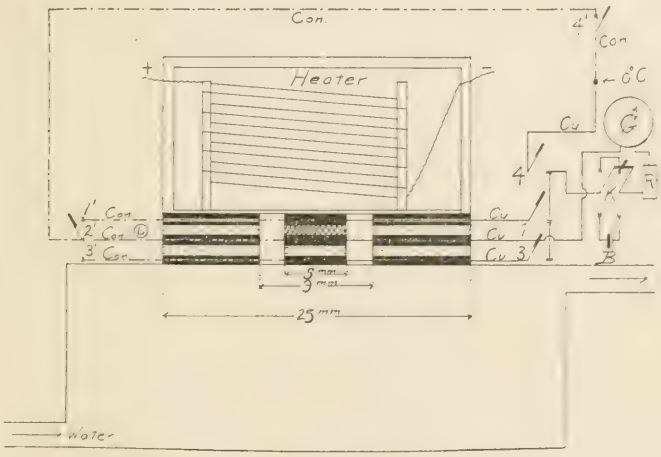

Fig. 1.

switches and other apparatus at the right clearly indicate the manipulations. When the key K is closed upward, the various thermo-couples can be thrown in series with the galvanometer so as to enable one to read the temperature difference between the middle junction No. 2 and any of the others, Nos. 1 or 3, or an outside couple, No. 4, the latter for purposes of calibration. For example if, with K closed upward, key No. 1 is closed and double throw key No. 2′ is connected with terminal, 1′, we have the thermo-couple copper-constantan in series with the galvanometer and the resistances R with the hot (usually) junction in the top copper disc, and the cooler junction in the middle disc. There is an electrical by pass in this circuit, viz., the conduction through the selenium crystal between discs No. 1 and 2, but considering that the whole resistance of the thermo-couple circuit, including the galvanometer, was only about 40 ohms and that the resistance of the crystal was over 1,000,000 ohms we see that no measurable error could enter. If all the keys are opened but No. 1, and K is closed downward, then a cell B is placed in circuit and the electrical conductivity of the crystal can be obtained by the galvanometer deflection. A 25-watt tungsten lamp placed at L could be thrown on or off at will. The light passed through

the clear glass of the upper washer, thus removing much of the heat energy, and so illuminated the selenium crystal. The illumination was sufficient to increase the equilibrium electrical conductivity at ordinary room temperature to nearly three times the dark conductivity.

The theory of the experiments rests on the assumption that the heat that enters the top copper disc passes on down through the pile of discs without loss and is ultimately delivered to the cooling water. In the original method Christiansen used three flat discs with imbedded thermometers and attempted to keep the middle copper disc at room temperature. In this improved guard ring method, if there is about the same temperature gradient between heater and cooler through the discs and the inside of the washers, then there should be no lateral gain or loss of heat so far as the central discs are concerned. If the temperature of the top copper disc is taken as T_1, of the middle one T_2 and of the bottom one T_3, if the thickness of the selenium crystal is d_1 and of the glass disc is d_2, if their thermal conductivities are represented by K_1 and K_2 respectively, and lastly if we assume that the quantity of heat H that enters the top disc likewise leaves the lowest one,

$$H = K_1 \frac{(T_1 - T_2)A \cdot t}{d_1} = K_2 \frac{(T_2 - T_3)A \cdot t}{d_2}.$$

Since the time of flow, t and the areas of the discs, A are the same, we get

$$\frac{K_1}{K_2} = \frac{T_2 - T_3}{T_1 - T_2} \cdot \frac{d_1}{d_2},$$

or with constant thickness of the two discs,

$$\frac{K_1}{K_2} = C \frac{T_2 - T_3}{T_1 - T_2},$$

where C is a constant.

We are thus able to obtain, not the absolute thermal conductivity of the selenium crystal, but rather its ratio to that of glass. For absolute measurements a correction would have to be made on account of the fact that we assume the temperatures of the copper plates to represent the true temperatures of the two sides of the crystal and of the glass respectively. Christiansen gives the necessary correction formulas.[1] In the present investigation no attempt has been made to obtain the absolute thermal conductivity of the selenium crystal. This is because the other question was of paramount interest, and because, on account of the small interstices of the crystal, one would have had to make careful corrections for the heat transmitted by radiation. However, the apparatus is extremely sensitive and reliable for these comparison measurements.

[1] Loc. cit.

The procedure was as follows: The tap water had been running for some hours until it had reached a steady temperature. The heater was left at room temperature in a room, the temperature of which held within a degree during any one set of experiments. After equilibrium had been attained, usually in ten to twenty minutes, the galvanometer readings for $T_1 - T_2$, $T_3 - T_2$, and $T_4 - T_2$ were obtained. T_4 was the temperature of the extra junction and was used for calibrating the

T_1-T_2	T_3-T_2	T_4-T_2	T_1	T_2	T_3	T_1+T_2 2	C	k_1/k_2 × Const.	R	State of Selenium.
10.9	− 3.3	6.7	27.2	16.3	13.0	21.8	27	.34	2.48	dark
10.9	− 3.4	6.3	27.6	16.7	13.3	22.1	67	.35	.....	light
12.5	− 4.1	3.7	31.8	19.3	15.2	25.6	33	.37	2.12	dark
12.8	− 4.2	3.7	32.1	19.3	15.1	25.7	70	.37	.....	light
19.4	− 7.1	− 2.3	44.7	25.3	18.2	35.0	41	.39	1.75	dark
19.5	− 7.2	− 2.6	45.1	25.6	18.4	35.4	72	.39	.....	light
27.9	− 9.8	− 7.7	58.6	30.7	20.9	44.7	45	.36	1.67	dark
28.2	−10.4	− 7.7	58.9	30.7	20.3	44.8	75	.38	.....	light
46.1	−17.5	−19.8	88.9	42.8	25.3	65.9	53	.39	1.55	dark
46.4	−17.5	−19.2	88.6	42.2	24.7	65.4	82	.38	.....	light
61.7	−23.8	−28.8	113.5	51.8	28.0	82.7	92	.39	1.28	light
61.7	−24.4	−29.4	114.1	52.4	28.0	82.3	72	.40	.....	dark
73.8	−29.1	−37.4	134.2	60.4	31.3	97.3	72	.40	.....	dark

couples so that galvanometer readings could be reduced to degrees Centigrade. All these readings were taken in a dimly lighted room. The electrical conductivity of the selenium was then obtained by closing key No. 1 and key K downward. The lamp L was now lighted, and after about twenty minutes similar readings were again made. While these readings were being reduced a small current was turned on through the heater. After equilibrium the process was repeated, and so on with various temperature gradients. Typical results of such a set of experiments are found in the table. The galvanometer deflections are for brevity not included. The first six columns are self-explanatory. The seventh represents roughly the mean temperature of the selenium. The column headed C gives the galvanometer deflections and hence a measure of the electrical conductivity of the selenium. The column headed $k_1/k_2 \times$ constant, gives a measure of the relative conductivity of the selenium as compared with the glass. The latter has an unknown, but undoubtedly small temperature coefficient. R is the ratio of the electrical conductivity of the selenium in the light to that in the dark. The last column is self-explanatory. Further examination of the table will make it clear that the ratio k_1/k_2 is not modified by changing from the light to the dark for any given temperature. This means of course, if we assume the light to have no appreciable effect on the thermal

conductivity of the glass, that it has no appreciable effect on the thermal conductivity of the selenium. There seems to be a slight increase in the ratio of k_1/k_2 wtih higher temperatures, but whether this is due to a decrease of the conductivity of the glass used, or an increase in the conductivity of the selenium, or both is impossible to state. If one had at his command a glass, the temperature coefficients of which were known, then he could settle this point.[1]

Incidentally, although the investigation was not directed toward this point, one notes that the electrical conductivity of the selenium both in the dark and in the light increases with increased temperature, but that the ratio of the two decreases. In other words, the selenium while becoming a better conductor of electricity is becoming less light sensitive.

Conclusion.—There may be advanced perhaps a number of reasons why the thermal conductivity was not appreciably changed by the illumination. One might suppose that the number of free electrons is so very small (judging from rough estimates of specific conductivity, about 10^{-12} the number present in copper) that even if there were a threefold increase in their number by the action of the light, that still the number is too small to make an appreciable effect in the thermal conductivity. In other words, the heat conduction in a selenium crystal is largely a matter of the vibrating atoms and molecules.

Another possibility is to assume that the number of free electrons present is not solely determined by the state of light or dark, but is more largely determined by the electric field intensity. If this were so, then one could easily see how the electrical conductivity could be increased without a corresponding increase in the thermal conductivity. The electrons could be assumed to be in an unstable state within the atom, and under the action of the light the instability is increased, but still the electrons are not expelled until an electromotive force is applied. The fact that the resistance of these crystals, and also of selenium cells in general is lowered by increased potential is evidence along this line. The lowered resistance can be interpreted as representing more actually freed electrons.

Whatever the explanation, the experimental results developed that whilst the given source of light increased the electrical conductivity nearly 300 per cent., the same light did not increase the thermal conductivity in any appreciable manner. Considering all possible sources of error, it must certainly have been under five per cent.

PHYSICS LABORATORY,
UNIVERSITY OF IOWA.

[1] The glass used was supplied by Bausch and Lomb from their regular cover-glass stock.

[Reprinted from the PHYSICAL REVIEW, N.S., Vol. VI., No. 4, October, 1915.]

THE CHANGE IN THE ELASTICITY OF ALUMINUM WIRE WITH CURRENT AND EXTERNAL HEATING.

BY H. L. DODGE.

THIS paper is the third of a series upon the effect of temperature on the elasticity of wires and deals with aluminum. The apparatus is the same as that used in work with copper and mild steel wires already described in the PHYSICAL REVIEW.[1] Upon these metals very exhaustive tests were made, the necessity of this being evident from the great variation in the results that had been reported. The treatment of the specimens of aluminum and the methods employed to secure a wide variation in the conditions have been the same as are fully described in former papers. In general the results have been the same. Consequently this paper is restricted to a brief discussion of the effects peculiar to aluminum and to a discussion of certain general conclusions regarding the effect of temperature on the elasticity of metals.

The tests have been made upon two samples of aluminum wire known commercially as "soft" and "hard." The former is annealed wire and bends easily, the latter is unannealed and is stronger and stiffer. The wires and their chemical analyses were secured through the kindness of Mr. H. M. Hall, superintendent of the United States Aluminum Co., Massena, N. Y. Below are tabulated the analyses, certain physical constants and data regarding the tests.

Chemical Analyses and Constants.

	Soft Aluminum.	Hard Aluminum.
Iron	.44 per cent.	.32 per cent.
Silicon	.24 " "	.22 " "
Copper	.03 " "	.07 " "
Aluminum	99.29 " "	99.39 " "
Diameter	1.24 mm.	1.44 mm.
Breaking load	22. kg.	38. kg.
Coefficient of expansion	.000025	.000025
Length	57.7 cm.	57.5 cm.
Permanent load	1,114. g.	1,114. g.
Added load	1,048. g.	1,048. g.
Total load per sq. mm.	1.8 kg.	1.3 kg.
Maximum current	23. amp.	
Maximum current per sq. mm.	19.1 amp.	

[1] PHYS. REV., N. S., Vol. 2, 431, 1913; Vol. 5, 373, 1915.

Aluminum will not stand very heavy loads. The percentage accuracy of the determinations of Young's modulus is consequently not as high as has been the case when the loading and resulting total stretch were considerably greater. The curves may however be regarded as giving the change in the modulus to an accuracy of one per cent. The error in the measurement of the temperature is of course greatest at high temperatures but probably does not exceed ten or at the most fifteen degrees.

TESTS UPON SOFT ALUMINUM WIRE.

The specimen of soft aluminum wire was first subjected to loads of two and four kilograms and a temperature of about 250° C. This treatment straightened out the kinks and lengthened the wire several millimeters. The weights were then made of appropriate sizes and two series of readings were taken with external heating. Series 2, Fig. 1,

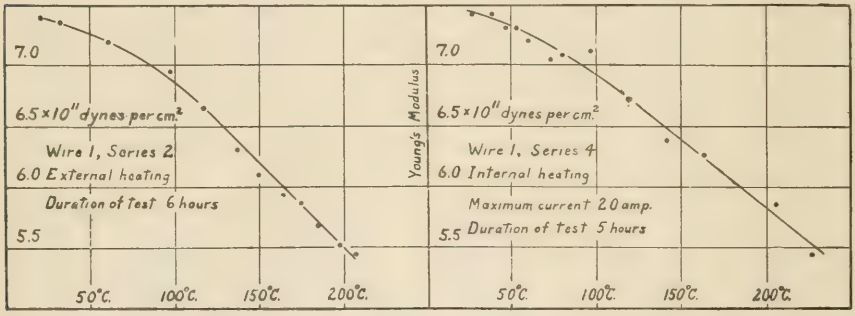

Fig. 1.

Effect of current and external heating upon the Young's modulus of a soft aluminum wire.

shows how the rate of decrease of the modulus increased rapidly with increase of temperature until it became uniform at about 100° C. The third series was with decreasing temperature and was similar in character. Series 4, Fig. 1, differs only in the manner of heating, an electric current in the wire itself being used.[1]

A comparison of these results with those shown in series 5, Fig. 2, shows that a permanent change in the properties of the specimen resulted from the treatment in test 4. In this test the temperature reached was higher than before, resulting in some stretching of the wire. The annealing was consequently more thorough. Series 5 was with external heating, increasing and decreasing temperature, and may be regarded as giving the change of Young's modulus with temperature for a thoroughly annealed aluminum wire.

[1] In all figures dots represent observations with increasing temperature, crosses those with decreasing temperature.

Series 6 was taken in the same manner except that the heating was by a current in the wire itself. The curves in the two drawings are practically the same. They show that a cyclic state had been reached in which Young's modulus becomes a function of temperature and is

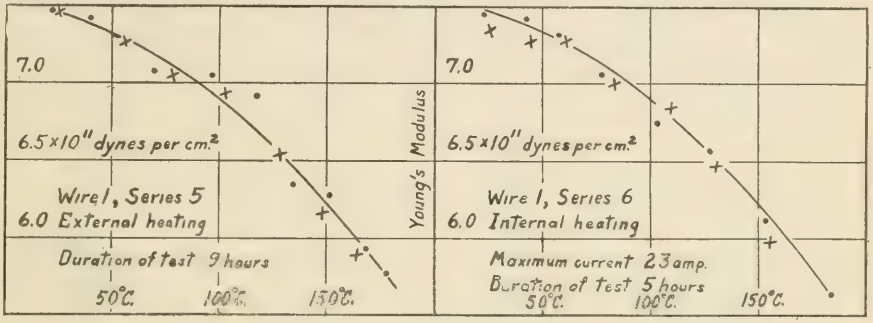

Fig. 2.

Effect of current and external heating upon the Young's modulus of a soft aluminum wire after thorough annealing.

independent of the manner of heating and the thermal route by which any temperature is reached.

Table I. gives the complete data from which series 6 is plotted. This is characteristic of all the data, except that with external heating a longer time was necessary for temperature equilibrium to be reached.

Tests upon Hard Aluminum Wire.

The sample of hard aluminum wire was carefully straightened and then tested without any preliminary annealing. Series 1, Fig. 3, shows the almost linear relation between the modulus and temperature. This

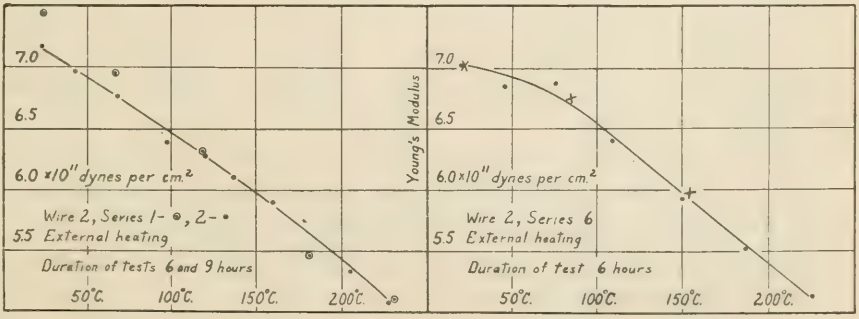

Fig. 3.

Effect of temperature upon the Young's modulus of a hard aluminum wire.

changes slightly with the second heating as shown by the curve of series 2. The next few series of observations were taken under a variety

of conditions, some with heavier weights and at higher temperatures. Just before series 6 the wire was heated to about 400° C. This treatment resulted in a partial annealing and produced the change in the properties of the wire shown by Fig. 3. The similarity of these results to those secured with the soft aluminum specimen lead me to believe that further testing with the consequent thorough annealing would have developed the same final conditions as before.

TABLE I.

Data for Soft Aluminum Wire, Series 6, Internal Heating.

Obs. No.	Time.	Temp.	Stretch.	Current.	No. of Obs.	Young's Modulus $\times 10^{-11}$, Dynes per cm.^{2}
1	9:00 A.M.	22° C.	.0658 mm.	0 amp.	9	74.5
2	10:00	47	.0660	8	10	74.2
3	10:20	57	.0670	11	9	73.1
4	10:35	78	.0694	14	8	70.6
5	10:50	102	.0726	17	10	67.5
6	11:00	128	.0745	19	11	65.8
7	11:15	154	.0800	21	6	61.2
8	11:25	183	.0870	23	23	56.4
9	11:30	156	.0820	21	7	59.8
10	11:40	131	.0758	19	13	64.6
11	11:50	110	.0717	17	10	68.4
12	12:05 P.M.	83	.0700	14	9	70.0
13	1:00	61	.0673	11	9	72.9
14	1:20	44	.0674	8	11	72.8
15	1:45	25	.0668	0	10	73.4

The results of the tests upon samples of soft and hard aluminum wire may be summarized as follows:

1. Aluminum wire can by thorough annealing be brought to a cyclic condition or steady elastic state in which Young's modulus becomes a function of temperature.[1]

2. Within given temperature limits the modulus becomes practically independent of history, the thermal route by which any temperature is reached having no apparent effect on the value of the modulus.

3. Heating by an electric current has no effect other than that caused by the accompanying temperature.

4. The Young's modulus of annealed wire decreases with increase of

[1] Guthe and Sieg, PHYS. REV., Vol. 30, 610, 1910, and Sieg, PHYS. REV., Vol. 31, 421, 1910, in studying the elastic constants of platinum-iridium wires by means of torsional vibrations found that annealing would bring the wires to such a state that results were easily reproducable.

temperature at an increasing rate. The following table is compiled from the results for thoroughly annealed soft aluminum wire.[1]

TABLE II.

Change of Young's Modulus of a Thoroughly Annealed Aluminum Wire with Increase of Temperature.

Temperature.	Young's Modulus $\times 10^{-11}$, Dynes per cm.^{2}	$\frac{dE \times 10^{-11}}{dT}$	Total Decrease, Per Cent. of E at 20° C.
20° C.	7.50	.0046	0.0
40	7.40	.0057	1.3
60	7.27	.0072	3.1
80	7.10	.0090	5.3
100	6.90	.0112	8.0
120	6.66	.0135	11.2
140	6.38	.0155	14.9
160	6.06	.0165	19.2
180	5.72	.0170	23.8

5. Hard aluminum wire exhibits an almost constant temperature coefficient of elasticity. Annealing does not materially change the value of Young's modulus but it does however affect the rate of change, increasing the temperature coefficient of elasticity for low temperatures and decreasing it for higher values, the dividing temperature being about 100° C.

General Conclusions Regarding the Effect of Temperature upon the Young's Modulus of Metals.

When the present series of investigations on the effect of current and external heating upon the Young's modulus of metals was undertaken the literature of the subject revealed great inconsistency and lack of uniformity in the results that had been secured. Although most observers had reported a decrease of elasticity with increase of temperature many did not find this to be the case. So great were the variations in the results that no conclusion could be drawn regarding the true nature of the effect of temperature on Young's modulus.

Two explanations were possible. Either every specimen possessed its own peculiar elastic properties or else the apparatus and methods employed were not sufficiently refined for the very delicate measurements required.

[1] Slotte, Acta. Soc. Scien. Fennicæ, Vol. 26, 1899, for a temperature range of 6° C. to 70° C. reports an almost uniform decrease in the modulus of .16 per cent. per degree. Katzenelsohn, Diss. Berlin, 1887, reports a decrease of .19 per cent. per degree between 0° C. and 100° C.

It was my belief that samples of wire ought by the proper heat treatment to be brought to a condition in which the modulus would be a function of temperature. With this as a criterion the apparatus described in previous papers was developed. The results secured with the cruder forms at once convinced me that a great part at least of the inconsistencies in former work must have been due to poor temperature control and the intrusion of certain factors affecting the accuracy of the determination of the modulus. The greatest difficulties were met when the heating was by an electric current in the wire itself as might have been anticipated from the fact that no satisfactory investigations had ever been carried out with this method of heating.

Copper was chosen for the first tests because of its high electrical conductivity and with the idea that it ought to respond readily to moderate heat treatment. My first paper[1] describes how the copper wire finally reached a cyclic state in which the Young's modulus became a function of temperature, the modulus decreasing at an increasing rate. The experience gained in the study of this metal lead me to believe that there was no reason why other metals should not behave in a similar manner. If any metal were to put this idea to test it seemed as if it would be one possessing such variable and peculiar properties as iron or steel. The second paper[1] describes the results with a mild steel wire. A cyclic state was secured for a temperature range of 20° C. to 475° C. and the general nature of the change of the modulus with temperature proved to be the same as in the case of copper.

Since these experiments were performed almost identical results have been reported by Lea and Crowther[2] with rods of mild steel, nicro-copper, and high-tension brass and by Harrison[3] with nickel wire, although in the latter case anomalous effects occur near the critical temperature. Similar results are also to be found in the work of some of the earlier investigators.

In the present paper aluminum has been shown to yield exactly the same type of results as were secured with copper and mild steel. A considerable amount of data is now available and for the metals that have been mentioned the following generalizations appear to be justified:

1. By thorough annealing the metal can be brought to a cyclic state in which Young's modulus becomes a function of the temperature.

2. The Young's modulus of the metal decreases with increase of temperature, the rate of the change increasing with the temperature.

[1] *Loc. cit.*

[2] Engineering, Vol. 98, 487, 1914.

[3] Phys. Soc., Proc., Vol. 27, 8, 1914.

3. When the metal is heated by the passage of an electric current the change in the modulus is a pure temperature effect.

In my own mind it seems so reasonable to expect that other metals will behave in the same manner that I am led to predict that these generalizations will be found to be the laws governing the effect of temperature upon the elasticity of metals.[1] Work is being continued in this laboratory with other metals and it is to be hoped that other investigators will continue with different types of apparatus and under different conditions.

Physical Laboratory,
State University of Iowa,
May, 1915.

[1] Dodge, Phys. Rev., N. S., Vol. 5, 76, 1915.

[Reprinted from the PHYSICAL REVIEW, N. S., Vol. VII, No. 4, April, 1916.]

CERTAIN CASES OF THE VARIATION OF SOUND INTENSITY WITH DISTANCE.

BY G. W. STEWART.

IT is not possible to secure a source of sound without the presence of reflecting surfaces. In quantitative considerations one desires to know the variation of intensity with distance from the source and is thus led to inquire as to the deviations from the inverse square law caused by the reflectors. A case of practical interest is that in which a small vibrating area is located on a rigid sphere, for it is possible to construct a source which conforms very closely to these theoretical conditions.[1] An additional interest arises from the fact that an investigation of such a source leads to an estimate of the deviations from the inverse square law in the case of a person speaking or singing. Furthermore, by utilizing the results of the same theoretical investigation, we can obtain the relative intensities on a sphere when the distance of the sphere from a simple source is varied. These values give an estimate of the deviations from the inverse square law in the case of a person listening. The deviations in the case of a speaker are of interest in architectural acoustics: those occurring with the listener are of importance in the psychological laboratory.

With a small vibrating area located on a rigid sphere, the geometrical figure of the reflector makes a mathematical investigation possible. Lord Rayleigh[2] was the first to obtain an expression for the sound intensity in the various directions at a great distance from such a sphere. Subsequently the writer[3] extended the investigation in order to obtain similar results for distances that are not great.

A brief statement of the theory will doubtless prove helpful to the reader. Let the source be confined to a small area on the surface of the sphere within which $P_n(\mu)$ of Legendre's series approximates unity. Assume that the velocity of this area is represented by Ue^{ikat}, and that it has the same magnitude at all points. Assume that

ψ is the velocity potential,

a is the velocity of sound,

[1] Stewart and Stiles, PHYS. REV., N. S., Vol. I., No. 4, 1913, p. 309, and PHYS. REV., N. S., Vol. III., No. 4, 1914, p. 256.

[2] Rayleigh, Theory of Sound, Vol. II., p. 254.

[3] Stewart, PHYS. REV., XXXIII., No. 6, p. 467, December, 1911.

r is the distance from the center of the sphere,
c is the radius of the sphere,
dS is an element of surface of the sphere,
k is 2π/wave length,
γ is $k(at - r + c)$,

F is $\Sigma \frac{2n+1}{2} P_n(\mu) \frac{\alpha\alpha' + \beta\beta'}{\alpha^2 + \beta^2}$,

and that G is $\Sigma \frac{2n+1}{2} P_n(\mu) \frac{\alpha\beta' - \alpha'\beta}{\alpha^2 + \beta^2}$,

where $f_n^1(ikr) = \alpha' + i\beta'$
and $F_n^1(ikc) = \alpha + i\beta$.

Then $\dot{\psi} = ka/2\pi r(F \sin \gamma + G \cos \gamma)\int\int Uds$. The mean potential energy, which is the "intensity" we desire, proves to be

$$\tfrac{1}{2}\rho_0 \frac{\dot{\psi}^2}{a^2} = \tfrac{1}{2}\rho_0(F^2 + G^2)\left(\frac{k}{2\pi r}\int\int UdS\right)^2.$$

In this formula $\left(\int\int UdS\right)^2$ measures the intensity of the source. For a constant source, a constant sound velocity and a fixed wave-length, the relative intensities are proportional to

$$\frac{(F^2 + G^2)}{r^2}. \qquad (1)$$

F and G are functions of k, r, c, and $\cos\theta$ or μ. The accompanying Fig. 1 will make clear the meaning of r, c and $\cos\theta$. The source is located at the point $(c, \theta = 0°)$. P is the point at which the intensity is desired and is located at a distance r and in the direction θ. Obviously P is any point on a circle whose radius is $r \sin\theta$ and whose circumference is everywhere at a distance r from the center of the sphere.

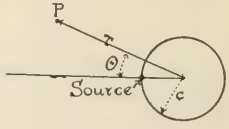

Fig. 1.

In view of (1), the deviations of the intensities from the inverse square law are indicated by relative values of $F^2 + G^2$. The computations involved in securing the numerical values of $F^2 + G^2$ are very laborious. Fortunately, certain numerical results are available, these having been obtained in previous investigations. As already indicated, the values of F and G depend upon those of ikr and ikc,[2] as well as upon μ.

[1] $f_n(ikr)$ and $F_n(ikc)$ are defined in Rayleigh's Theory of Sound, Vol. II., p. 238, and by Stewart, loc. cit.

[2] For the terms from which F and G can be computed in the two cases $kc = 0.5$, $kr = 25$ and $kc = 1.0$, $kr = 50$, see PHYS. REV., XXXIII., No. 6 (1911), Table I., p. 473, and Table II., p. 475.

Table I. indicates the variation in the relative values of $F^2 + G$ with r and with θ, the former being expressed in terms of c, the radius of the sphere. In order to show the percentage deviations in $F^2 + G^2$ which represent the percentage deviations from the inverse square law, the value of this sum is assumed to be unity at a distance of 50 c. Table I. utilizes the values of F and G when $k \times c$ is 1 and when $k \times r$ has the values 2, 3, 4 and 50. The values enclosed in parentheses are computed by substituting in each case for the distance r in (1) the distance from P the source on the sphere. In other words, the values in parentheses

TABLE I.

Relative Values of $(F^2 + G^2)$, those at 50 c being assumed Unity.

Radius of sphere, $c = \frac{\text{wave length}}{2\pi}$.

Distance.	$\theta = 0°$.	30°.	60°.	90°.	120°.	150°.	180°.
2 c	4.24 (1.10)	2.58 (.98)	1.14	.622	.396	.329	.270
3 c	2.26 (1.05)	1.81 (.98)	1.18	.728	.564	.446	.420
4 c	1.75 (1.03)	1.53 (.98)	1.15	.862	.661	.544	.520
50 c	1.00 (1.00)	1.00 (1.00)	1.00	1.00	1.00	1.00	1.00

indicate the deviations from the inverse square law when the distances are measured from the actual source rather than from the center of the sphere.

TABLE II.

Relative Values of $(F^2 + G^2)$, Those of 200 c Being Assumed Unity.

Radius of sphere $= 0.5 \times \frac{\text{wave-length}}{2\pi}$.

Distance.	$\theta = 0°$.	30°.	60°.	90°.	120°.	150°.	180°.
2 c	6.28 (1.59)	3.54 (1.30)	1.28	.652	.655	.765	.797
50 c	1.04 (1.01)	1.03 (1.01)	1.02	1.00	.978	.968	.957
200 c	1.00 (1.00)	1.00 (1.00)	1.00	1.00	1.00	1.00	1.00

Table II. contains the values obtained when $k \times c$ is 0.5 and when $k \times r$ has the values 1, 25 and 100.

The tabulated results show that when distances are measured from the center of the sphere the direction of minimum variation from the inverse square law is found to depend upon the wave-length. With the longer wave there is less deviation in the rear of the sphere and with the

For values of F and G for $kc = 1$ and $kr = 2, 3, 4$ and 50 see PHYS. REV., N. S., IV., No. 3, Table I., p. 255, and Table II. (60 cm.), p. 256.

The values of $F^2 + G^2$ for $kc = 0.5$ and $kr = 1.0$ and $kr = 100$ appear only in the form of curves, Fig. 2, PHYS. REV., XXXIII., No. 6, December, 1911.

shorter, less deviation in front. Both of these facts are in accord with anticipation based upon elementary considerations. If the distances in the directions 0° and 30° are measured from the source on the sphere there is less deviation from the inverse square law, but it is yet large. Whether distances be measured from the source or from the center of the sphere, the differences obtained by changing wave-length are very marked. This suggests that, in any practical case, a closer approximation to the inverse square law in front of the source can be secured by increasing the frequency of the tone. It also suggests the well-known fact that the relative intensities of the components of any sound will change with distance and direction from the source.

In order to utilize these numerical values for an estimate of the deviation from the inverse square law in the case of a speaker, it is necessary to assume that the head acts as a rigid sphere and that the source occupies but a small area on that sphere. If we choose as a circumference, 60 centimeters, the results in Table I. and Table II. refer to wave-lengths of 60 and 120 centimeters respectively. Although the lack of conformity to the theoretical conditions is obvious, yet we can consider that the above results furnish a fairly satisfactory estimate of the deviations from the inverse square law in the case of a speaker or singer using tones approximately 60 and 120 centimeters in wave-length, or approximately 575 and 287 in frequency. With the sphere circumference 60 centimeters, the distances are approximately 19.1, 28.6, 38.2 and 477 centimeters in Table I., and 19.1, 477 and 1,910 centimeters in Table II. In the former table, with a frequency 575, the deviations from the inverse square law at the above distances are over 400 per cent. If the distance to the source of sound instead of to the center of the head is substituted in the inverse square formula, then the deviations are 10 per cent. or less. The deviations in various directions are readily read in the tabulation. Table II. should be used for a frequency approximating 287. Neither table gives the relations between intensities at points having a constant r and varying values of θ, for such a comparison has already been published.[1]

In order to apply the results to the case of hearing, we must have recourse to the reciprocal theorem of Helmholtz,[2] which permits us to interchange positions of the source and the points at distances r where the relative intensities are desired. Then we can consider a simple source at a distance r from the center of the head (or, using the parenthetical results, from the ear) and can obtain an estimate of the variation of intensity with distance from either ear. For the frequency 575, the maximum deviation

[1] Stewart, PHYS. REV., XXXIII., No. 6, December, 1911.

[2] Rayleigh, Theory of Sound, Vol. II., p. 294.

from the inverse square law with distances from the center of the head to the source of approximately 19.1 and 477 centimeters (9.5 to 468 centimeters from the ear) does not exceed 10 per cent. (see Table I., 0°). For the frequency 287 and the same distance from the ear, the deviation is almost 60 per cent. (see Table II., 0°). If with the latter frequency distances of 477 to 1,910 centimeters are selected, the deviation is only 1 per cent. There is a distinct advantage in using the higher frequency in cases where an estimate of relative intensities is obtained by using the inverse square law.

Physical Laboratory,
State University of Iowa.

[Reprinted from the PHYSICAL REVIEW, N.S., Vol. VII. No 5, May, 1916]

THE EFFECT OF TEMPERATURE ON THE RESISTANCE, THE LIGHT-SENSITIVENESS, AND THE RATE OF RECOVERY OF CERTAIN CRYSTALS OF METALLIC SELENIUM.

BY KATHRYN JOHNSTONE DIETERICH.

THE purpose of this article is to set forth certain facts concerning the effect of variation of temperature on the rate of recovery after illumination, the sensitiveness to light, and the resistance of certain crystals of metallic selenium.

APPARATUS.

The apparatus used is shown in Fig. 1. The crystal was placed between platinum electrodes, E, such that the weight, W, suspended from the upper electrode, maintained a constant pressure on the crystal.

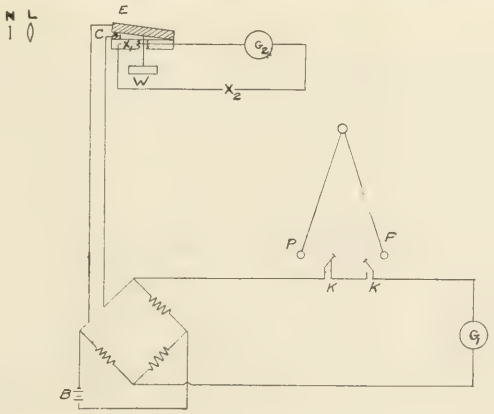

Fig. 1.

The source of illumination was a Nernst glower, N, at a distance of 45 cm., the intensity of illumination being kept constant by maintaining over the lamp a voltage of 115 volts from storage batteries. The crystal constituted one arm of a Wheatstone bridge circuit, over which a constant voltage of 10 volts was kept. As nearly as possible all conditions of light intensity, pressure, voltage, and time of recovery were kept constant and only temperature varied. The latter was measured by means of a copper-constantin thermocouple in series with a galvanometer. One

junction, X_1, was placed beside the crystal, the other, X_2, was kept at 0° C.

In carrying out this investigation it was necessary, at each temperature, to measure the resistance of the selenium when it had reached equilibrium in the dark, in the light, and after it had been in the dark the desired short interval of time, viz., .02 sec. The first two measurements were made by the usual balanced bridge method, the last by the method of Brown and Clark.[1] This consisted in balancing the Wheatstone bridge circuit with the selenium in equilibrium in the light, and then opening the galvanometer circuit. A timing pendulum, which had been arranged to operate a metal shutter to cut off the light from the crystal and throw the galvanometer in circuit simultaneously, was then released. A deflection of the galvanometer resulted, due to the change in resistance of the crystal when the light was removed. The resistance of the crystal at the end of .02 sec. in the dark was determined, later, by substituting for the crystal a carbon resistance, which could be adjusted, until upon releasing the pendulum, the same galvanometer deflection was obtained as had been given by the crystal. The carbon resistance, since it was constant, was then measured by balancing the bridge circuit.

THEORETICAL.

When a crystal of metallic selenium has been exposed to light, and the light is suddenly removed, the conductivity of the crystal does not return instantly to the value of the equilibrium dark conductivity, but decreases rapidly at first, then more slowly, requiring several minutes or even hours to attain its final value, depending on the temperature. When a gas is exposed to a source of ionization, and the source is suddenly removed, the conductivity decreases rapidly at first, then more slowly, until equilibrium is restored. In gases the decrease in conductivity during the first short interval of time is ascribed to the recombination of the conducting electrons with the positive residues. It has been shown by Rutherford,[2] McClung,[3] McClelland[4] and more recently, Plimpton,[5] that if there are present n_1 negative ions and n_2 positive ions, the rate of recombination is directly proportional to $n_1 \times n_2$, that is

$$\frac{dN}{dt} = -\alpha n_1 \times n_2,$$

where α is a constant and is called the coefficient of recombination.

[1] PHYS. REV., Series 1, XXXIII., p. 53, 1911.
[2] Phil. Mag., Series 5, Vol. 44, p. 422, 1897; Vol. 47, p. 109, 1899.
[3] Phil. Mag., Series 6, Vol. 3, p. 283, 1902.
[4] Phil. Mag., Series 5, Vol. 46, p. 29, 1898.
[5] Phil. Mag., Series 6, Vol. 25, p. 65, 1913.

In order to study the effect of pressure, light intensity, and electromotive force on the rate of recovery in selenium, in cells and crystals, Professor Brown[1] assumes that for the first short interval of time after the light has been removed the decrease in conductivity of a selenium cell or crystal is due to the recombination of the conducting electrons with positive residues, and that this recombination takes place according to the same law as that which holds for gases. If the number of electrons taking part in the conduction is proportional to the conductivity then the rate of recombination may be calculated from the formula:

$$\alpha_1 = -\frac{\Delta C}{C^2 \Delta t},$$

where ΔC is the decrease in the conductivity during the time, Δt, that the crystal has been in the dark, C is the equilibrium value of the conductivity in the light, and α_1 is proportional to the coefficient of recombination. In order to obtain this result, Professor Brown found it necessary to make the following assumptions:

1. When the crystal is in equilibrium in the light or in the dark the conductivity is proportional to the number of negative electrons taking part in the conduction.

2. Recombination takes place according to the same law as in gases.

3. The rate of recombination is the same in the light as in the dark.

The above conception was verified approximately by varying the conductivity over a wide range. This was done by changing the intensity of illumination. Since the rate of recombination was found to be nearly proportional to the square of the conductivity it was assumed that the recombination followed the same law as in gases.

In this paper the results of an investigation of the effect of temperature on the rate of recovery of selenium crystals will include a discussion of the effect of temperature on the coefficient of recombination, calculated on the above theory.

RATE OF RECOVERY.

The crystals used were prepared by Dr. Brown[2] as described by him. They were of three kinds: the large hexagonal, the needle shaped acicular, and the monoclinic lamellar. In each case the current was passed through the crystal in a direction perpendicular to the direction of the long axis, and the crystal illuminated in a direction perpendicular to the direction of the current.

In general the coefficient of recombination was found to decrease with increase in temperature.

[1] PHYS. REV., Series 2, Vol. 5, p. 395, 1915.

[2] PHYS. REV., Series 2, Vol. 5, p. 236, 1915; Vol. 4, p. 85, 1914.

TABLE I.

Temperature in °C.	Temperature in °Abs.	Conductivity in Light. Ohms^{-1} × 10^{7}.	$\alpha \times 10^{-7}$.	$\alpha \times C$.
−89	184	1.685	3.640	6.11
−68	205	2.057	3.280	6.75
−40	233	3.580	1.680	6.02
0	273	7.070	0.875	6.17
19	292	7.460	0.835	6.22
21	294	7.880	0.754	5.93
26	299	8.070	0.730	5.89
33	306	8.660	0.606	5.23
43.5	316.5	9.070	0.548	4.95
56	329	9.310	0.537	4.99
64	337	5.890	0.367	2.17
75	348	8.320	0.316	2.62

Table I. shows the results obtained with hexagonal crystal No. 2. There is seen to be a large fall in the value of α with increase of temperature from − 89° C. to about 0° C., after which the decrease is more gradual to 75° C. which is as high a temperature as could be reached. Upon examining the electrodes it was discovered, early in the work, that at the higher temperatures a dark deposit was formed between the

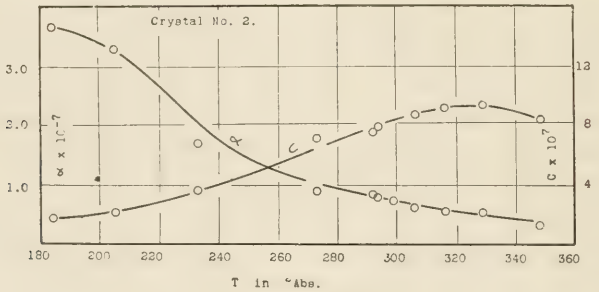

Fig. 2.

crystal and the electrodes. The effect of this deposit was to depress both the values of conductivity and of α, so that the last two points on the curve, Fig. 2, showing the relation between α and temperature, are only approximately correct.

Crystal No. 13, Table II., shows in general the same result; a large decrease in α with increased temperature. In this case the temperature was not high enough to produce sufficient deposit to cause any appreciable error.

The curve, Fig. 3, shows a slight depression in the values of α at 10° C., which may or may not be a genuine effect. However, two complete

series of readings gave a low value of α at just this temperature. It may be of interest to mention here that, although a crystal had been removed from the electrodes for several weeks, it could be returned and, if the

TABLE II.

Crystal No. 13. E.M.F. = 6 volts. Pressure = 75 gr.

Temperature in °C.	Temp rature in °Abs.	Conductivity in Light. Ohms^{-1} × 10^{7}.	$\alpha \times 10^{-7}$.	$\alpha \times C$.
−40	233	2.63	2.010	5.26
− 7	266	4.35	0.925	4.03
10	283	7.10	0.460	3.26
21	294	6.05	0.548	3.31
32	305	6.82	0.398	2.73
41	314	7.38	0.348	2.57

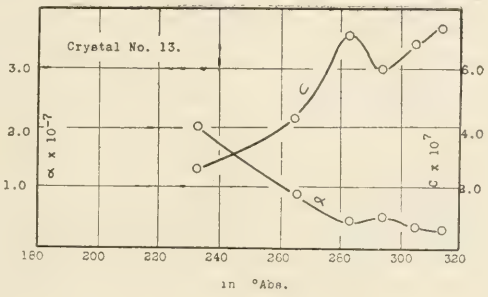

Fig. 3.

previous conditions as to light intensity, pressure, and voltage were restored, the same curve of α and temperature was obtained, giving the same values of α as before.

Other crystals showed essentially the same change of α with temperature. In some cases there were depressions as in No. 13; one crystal, No. 3, a lamellar type, showed quite a sharp kink at 35° C. Thus, each of the three types of crystals showed the same effect of temperature on the coefficient of recombination: a general decrease in α with increase of temperature.

H. A. Erikson,[1] in 1909, made a determination of the effect of temperature upon the coefficient of recombination in gases. He obtained a curve showing the relation between α and T which resembles very closely the curve obtained for crystal No. 2, Fig. 2. His curve is shown in Fig. 4. However, no definite significance can be attached to this striking similarity as a result of this investigation only.

[1] Phil. Mag., Series 6, Vol. 18, p. 328, 1909.

If then, the recovery of selenium crystals is due to a recombination of negative electrons with positive residues, and the rate of recombination is expressed by the formula,

$$\frac{\Delta C}{\Delta t} = -\alpha C^2,$$

the coefficient of recombination, α, varies with temperature.

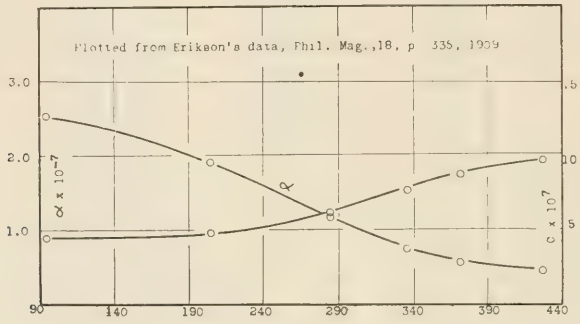

Fig. 4.

It will be noticed, in Tables I. and II., that, while the coefficient of recombination is not a constant at all temperatures, the product, $\alpha \times C$, is very nearly so. A glance at the curves, Figs. 2 and 3, will show that this relation suggests itself here also; since wherever a rise occurs in the α-curve a depression appears in the C-curve. The same general relation between the shape of the α-curve and the C-curve is apparent in Erikson's work in gases, Fig. 4. Dr. Brown[1] has found that $\alpha \times C$ is a constant for selenium crystals when pressure or voltage varied the conductivity, although α is not constant.

Since $\alpha \times C$ is a constant and is equal to $\Delta C/C \times \Delta t$, the rate of recovery during the first short interval of time is equal to some constant, β, times the conductivity in the light.

$$\Delta C/\Delta t = \alpha C^2 = \beta C;$$

where β is equal to $\alpha \times C$, and we have the simple relation that the rate of recovery of selenium crystals is proportional to the conductivity in the light. This proportionality did not appear to hold with much accuracy at the higher temperatures.

[1] *Loc. cit.*

RESISTANCE.

Conductivity is determined by taking the reciprocal of the resistance, so that a study of the effect of increase of temperature upon the conductivity of selenium crystals should give an adequate idea of its effect on their resistance. In crystal No. 2, Fig. 2, it will be seen that the conductivity in the light at low temperatures was small, increased with increasing temperature, and passed through a maximum in the region of 55° C. The fall of conductivity above 55° C. might have been influenced by oxidation of the brass electrodes which were used with this crystal, so that the apparent decrease was somewhat exaggerated. Crystal No. 13, Fig. 3, showed a general increase of conductivity in the light with increased temperature. There was, however, a decrease for a short interval from 10° C. to 21° C., after which the conductivity rose again as the temperature increased. Depressions of this kind were found in the *C*-curves of many of the crystals, although not always at this same temperature. One crystal, No. 4, had a sharp depression and sudden rise in the region of 0° C. Ries[1] found the same condition to hold for selenium cells, with the minimum most often around 100° C., but that he was able to shift the position of this minimum to below 0° C. by certain heat treatment. The general tendency, however, in cells and crystals is for an increase of conductivity, and a consequent decrease of resistance, to result from an increase in temperature.

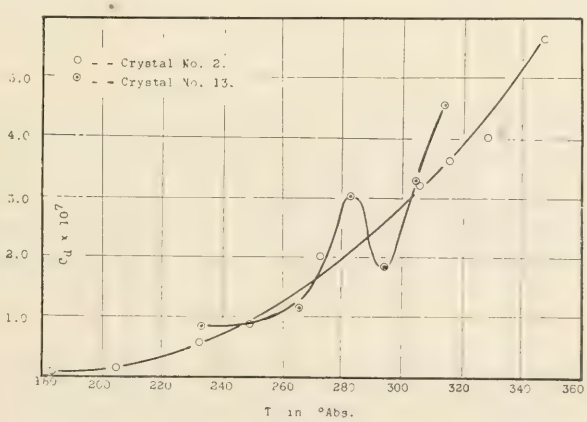

Fig. 5.

The equilibrium dark conductivity will be seen from the curves in Fig. 5 to follow much the same general tendency: an increase of conductivity with increase of temperature. The same maxima and minima which were found in the light conductivity curves are apparent here.

[1] Die Elektrischen Eigenschaften und die Bedeutung des Selens für die Electrotechnik, 1908.

LIGHT SENSITIVENESS.

The definition of light sensitiveness may be formulated arbitrarily to best meet the needs of practical use, as in the operation of galvanometers and relays, or it may arise from purely theoretical considerations arrived at in an attempt to explain the effect of light on crystals. Several of the more common expressions for light sensitiveness determined by practical usage are,

1. The ratio of the change of resistance to the resistance in the dark. $(R_d - R_l)/R_d$.

2. The ratio of the resistance in the light to the resistance in the dark. R_l/R_d.

3. The difference between the conductivity in equilibrium in the light and in the dark. $C_l - C_d$.

In the third case it is necessary to know not only this difference but also the terms which produce it, namely, C_l and C_d, to form an adequate idea of the possibility of adapting the crystal to the work at hand. For the same reason one would need to know the intensity of the light producing this change.

However, the theory which underlies this paper makes it reasonable to define sensibility in still another way, namely, in terms of the rate of production of conducting electrons. Dr. Brown[1] has shown how this rate of production may be calculated.

$$\frac{dC}{dt} = M + q;$$

where dC/dt is the rate of production of electrons when the crystal is in equilibrium in the light; q is the rate of production in the dark and M is the rate of production by the light. dC/dt must also be equal to the rate of recombination, αC^2, since the crystal is in equilibrium. For the first few hundredths of a second after the illumination is removed

$$dC/dt = \alpha C^2 = M + q.$$

When the crystal is in equilibrium in the dark the rate of production of electrons, q, is equal to the rate of recombination, αC_d^2, and M is zero. When the crystal is in equilibrium in the light

$$\Delta C/\Delta t = M + \alpha C_d^2;$$

$$M = \Delta C/\Delta t - \alpha C_d^2$$

$$= \alpha C_l^2 - \alpha C_d^2.$$

[1] *Loc. cit.*

TABLE III.

Crystal No. 2, in ° Abs.	C_l (Ohms^{-1} × 10^7.)	C(Ohms^{-1} × 10^7.)	$C_l - C_d$ (Ohms^{-1} × 10^7.)	M
184	1.685	.106	1.58	10.2
205	2.057	.135	1.92	13.8
233	4.190	.583	3.61	26.8
273	7.070	2.150	4.82	32.47
293	7.880	2.280	5.60	34.92
299	8.070	3.030	5.04	35.37
306	8.660	3.230	5.43	39.10
316	9.070	3.620	5.45	38.00
326	9.310	4.080	5.23	37.80
348	8.320	5.670	2.65	11.60
Crystal No. 13.				
233	2.630	.588	2.042	12.21
266	4.350	1.150	3.20	16.25
283	7.100	3.030	4.07	19.08
294	6.050	1.830	4.22	18.17
305	6.820	3.280	3.54	14.29
314	7.380	4.550	2.83	11.80

Table III. and Fig. 6 show the effect of temperature upon light sensitiveness as defined by the practical working definition, $C_l - C_d$. It will be seen that the sensitiveness at first increases with increase of tempera-

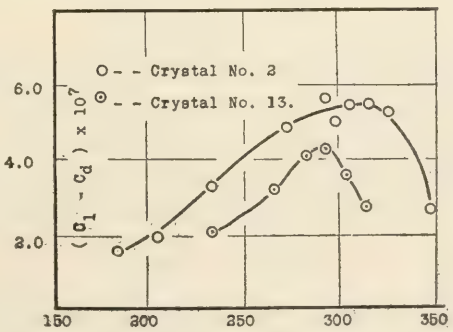

Fig. 6.

ture to about the region of room temperature after which it decreases. Prof. L. P. Sieg[1] has noted that selenium plates show a decrease in sensitiveness to light, from room temperature to 100° C. when the light sensitiveness is defined as the ratio of conductivity in the light to the

[1] PHYS. REV., Series 2, Vol. 6, p. 213, 1915.

conductivity in the dark. Fig. 7 shows the same general relation between temperature and light sensitiveness, when the latter is measured by the rate of production, M, of electrons by the light.

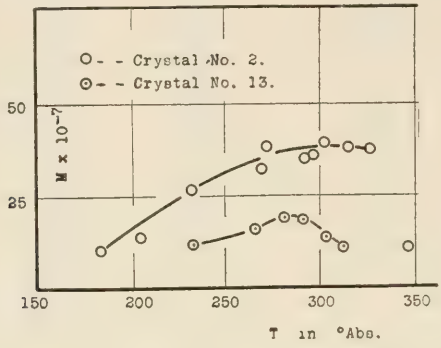

Fig. 7.

SUMMARY.

In conclusion, it may be said in regard to the effect of temperature on certain properties of selenium crystals, that,

1. The resistance in the light and in the dark is, in general, decreased by increase of temperature, with a small increase in some cases in the region 0° C. to 50° C.

2. The coefficient of recombination, α, calculated according to the formula,

$$\alpha = -\Delta C/C_1^2\Delta t,$$

is not constant at all temperatures but decreases with increase of temperature in much the same manner that the coefficient of recombination decreases in gases with increase in temperature.

3. Over a wide range of temperatures the rate of recovery for the first few hundredths of a second is proportional to the conductivity in the light.

$$\Delta C/\Delta t = -\beta C_1.$$

4. The sensitiveness to light does not vary greatly over a wide range of temperature, although a tendency toward a maximum is found in the region 30° C. to 50° C.

I wish to acknowledge my indebtedness to the staff of the Department of Physics for their interest in the problem, and especially to Dr. Brown for suggesting it, and for his encouragement during the progress of the work.

PHYSICAL LABORATORY,
THE STATE UNIVERSITY OF IOWA.

[Reprinted from the PHYSICAL REVIEW, N.S., Vol. VIII, No. 2, August, 1916]

THE EFFECT OF TEMPERATURE ON THE LIGHT-SENSIBILITY CURVES OF DIFFERENT TYPES OF SELENIUM CELLS.

BY E. O. DIETERICH.

THIS paper summarizes a study of the wave-length-sensibility curves of three types of selenium cells at temperatures higher than room temperature. The investigation was begun in the early fall of 1914 and some of the results reported at the November meeting of the American Physical Society that year. The experiments, however, were interrupted and it has not been possible to complete the work as originally planned. In the meantime an article describing the effect of low temperatures on the light-sensibility curves of selenium and stibnite cells has been published by D. S. Elliott[1] who finds a behavior similar to that described below. It was thought worth while, however, to call attention to these results inasmuch as they were obtained at high temperatures and on cells of entirely different nature than those used by Elliott.

The cells used were some prepared by the writer and previously described.[2] Three types were studied: Type A, which shows no maximum in the red whatever; type C, in which the maximum in the red is lower than that in the blue; type D, in which the maximum in the red is higher than that in the blue. The permanence of the light-sensibility curves of the different types was demonstrated here; those obtained at this time were essentially the same, for the same conditions of temperature, as those obtained when the cell was first completed. Fig. 1 shows these curves for a cell of type A.

To determine the light-sensibility curves, the method outlined by Brown and Sieg[3] and previously used was followed. For obtaining the curves at temperatures higher than room temperature the cell was placed in a small oven which was about one inch square and three inches deep. This oven was constructed of plaster of Paris, wound with Nichrome wire and insulated by means of asbestos packing. It was provided with a small mica window to admit the light. By connecting it to a battery of storage

[1] PHYS. REV., Series II., Vol. V., p. 53, 1915.
[2] PHYS. REV., Series II., Vol. IV., p. 467, 1914.
[3] PHYS. REV., Series II., Vol. II., p. 487, 1913.

cells a very constant temperature could be maintained. Temperatures were measured by means of a thermometer placed directly behind the cell. At the higher temperatures the cell served as a very sensitive means of indicating variations too small to be detected on the thermometer; the resistance changing to such an extent that the galvanometer deflec-

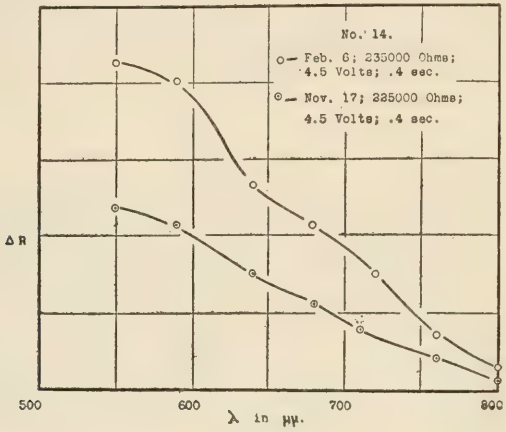

Fig. 1.

tion amounted to several hundred centimeters. Whenever possible, exposures of 0.5 sec. were taken, for with short exposures the fatigue of the cell is much less than with long exposures. However, at the higher temperatures the sensitiveness of the cells became so low that the short exposures gave galvanometer deflections too small to be read with very

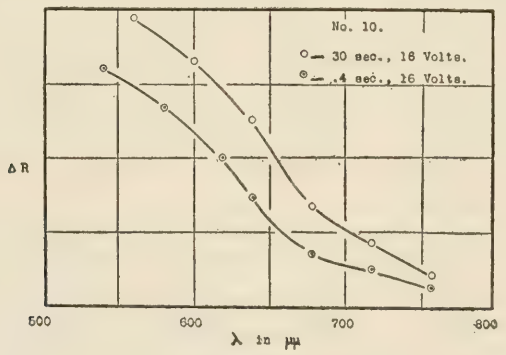

Fig. 2.

great accuracy, so here the exposures were increased in length. For certain types of cells, namely, type A, this change of period of illumination has no effect on the shape of the light-sensibility curve, as is shown in Fig. 2, where the time of illumination varies by a factor of 75. For those

cells whose light-sensibility curve depends upon the period of illumination a constant period of exposure, 10 sec., was used at all temperatures. At the higher temperatures, to increase the sensibility, higher voltages were also found necessary. It has been shown repeatedly, however, that a change of voltage does not alter the light-sensibility curves of selenium.[1]

The behavior of the various types of cells with increasing temperature is shown in Figs. 3, 4 and 5. Fig. 3 represents the curves obtained with

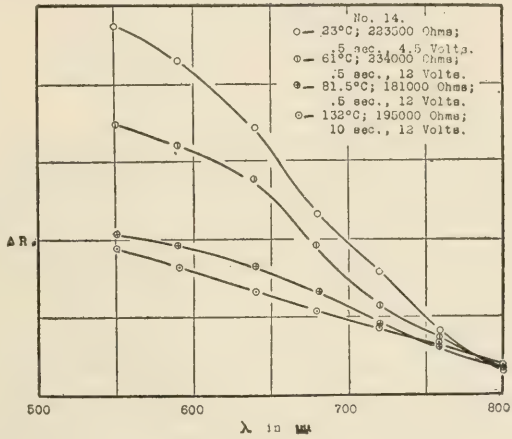

Fig. 3.

a cell of type A. The sensibility in the blue end of the spectrum decreases markedly while in the red it is hardly affected. At the highest temperature reached, however, there is no indication of an appearance of a maximum in the red, as might be expected, perhaps, as a result of the depression in the blue. Unfortunately, while trying to raise the tempera-

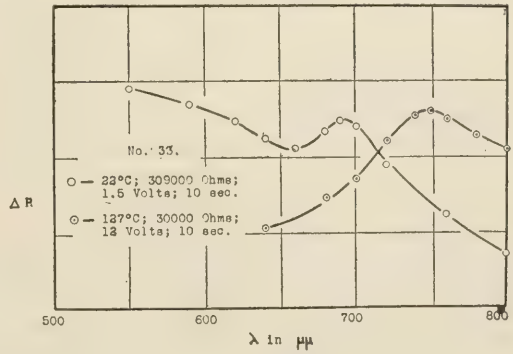

Fig. 4.

ture of this cell to somewhere near the melting point of selenium, the selenium melted and the identity of the cell was thereby destroyed.

[1] PHYS. REV., Series II., Vol. IV., p. 48, 1914.

From the behavior of cells of type A, one would be led to expect that in the case of cells with a maximum in the red, this maximum should be displaced towards the longer wave lengths. This was actually found to be the case. Figs. 4 and 5 show the progressive change of the position

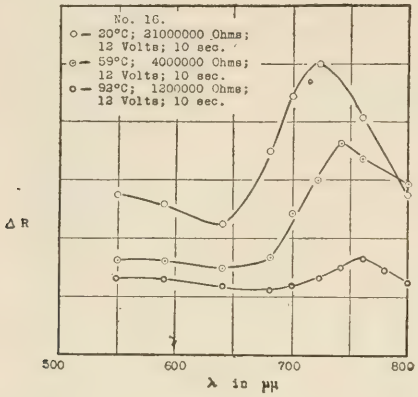

Fig. 5.

of maximum sensibility with increasing temperature. These results are in perfect agreement with those of Elliott[1] who finds the maximum shifted towards the shorter wave lengths at the temperature of liquid air.

Fig. 6 shows that the characteristic wave-length-sensibility curve at

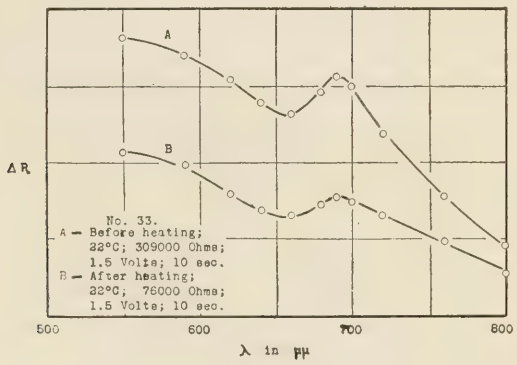

Fig. 6.

room temperature is not changed by the heat treatment; the maximum returns to the original place. The difference in resistance and sensitiveness before and after heating may probably be due to slow changes taking place in the selenium similar to those exhibited by the freshly manufactured cells.

PHYSICAL LABORATORY,
THE STATE UNIVERSITY OF IOWA.

[1] PHYS. REV., Series II., Vol. V., p. 53, 1915.

Reprinted from The Physical Review, N. S., Vol. VIII. No. 5, November, 1916.]

NOTES ON THE ELASTIC PROPERTIES OF PHOSPHOR-BRONZE WIRES.

By Arnold J. Oehler.

Introduction.

THE work by Guthe,[1] Guthe and Sieg,[2] and Sieg,[3] on platinum-iridium wires when used as suspensions for torsion pendulums, showed some remarkable elastic properties of that alloy. The principal one of these was the variation of the period with the amplitude of vibration. It was these studies that made it seem very desirable to test other alloys by a similar method, and because of the extensive use of phosphor-bronze wires as delicate suspensions, this work has been devoted to that alloy.

The wires employed in these experiments represented thirteen successive drawings from an original sample and ranged in diameter from .100 mm. to .508 mm. These wires were very kindly supplied by the American Electrical Works of Phillipsdale, Rhode Island.

Some work by Professor Sieg and the writer,[4] in 1914, showed that the periods of torsional vibrations of these wires were not constant but varied widely with the different amplitudes. The problem of this work was to study more carefully, the elastic nature of this alloy.

The apparatus was the same one employed and described by Sieg,[5] and the method of timing the vibrations was also essentially the same as in that work. The length of the suspension most commonly used was 30 cm. and the initial twist was, in most cases, ten degrees per cm. length of the suspension.

The whole problem of the elasticity of this alloy is very complicated and involved and many of the points have been carefully investigated. However, only a few of the most striking points will be discussed in this paper.

[1] K. E. Guthe, Ia. Acad. Sci., 15, p. 147, 1908. Abst. in Phys. Rev., 26, p. 201, 1908.
[2] K. E. Guthe and L. P. Sieg, Phys. Rev., 30, p. 610, 1910.
[3] L. P. Sieg, Phys. Rev., 31, No. 4, p. 421, 1910.
[4] L. P. Sieg and A. J. Oehler, Ia. Acad. Sci., 22, p. 321, 1915.
[5] L. P. Sieg, *loc. cit.*

DISCUSSION OF RESULTS.

The Three States of the Wires.—The first experiments soon showed that there were three distinct states, from one to another of which, the wires would change. This is most readily seen from the three typical period-amplitude curves shown in Fig. 1.

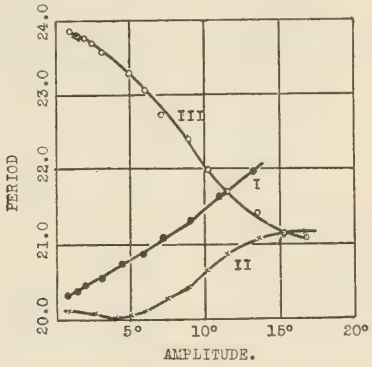

Fig. 1.

Curve *I.* represents state I. in which the period of vibration decreases with the fall in amplitude.

Curve *II.* represents state II. and is similar to curve *I* in the larger amplitudes but makes a departure from that type at an amplitude of about 4 degrees per cm. length of the wire,[1] and from that time on, the period increases as the amplitude decreases.

Curve *III.*, representing state III., shows the greatest departure from a constant period and indicates a continual increase in period as the amplitude decreases.

Of these three states, II. and III. were the most common and state I. was easily changed into state II. There were several cases in which the period was essentially constant below an amplitude of 4 degrees per cm. length of the wire, and it thus appears that type I. is closely connected up with type II. Types II. and III. are very distinct, however, and no evidence has been obtained that would show an intermediate state between the two. When there was change of state the change was always complete from one to the other.

The magnitude of this variation of the period with the amplitude is best seen by considering the curves of Fig. 1 in detail. The three

[1] By amplitude it shall be understood that we mean the complete angular displacement of the pendulum between its extreme positions of vibration. Thus we really have double amplitudes and the above minimum really occurs at an amplitude of about 2 degrees per cm. length of the suspension.

Also the periods mentioned in this paper are, strictly speaking, only half-periods. The method of experimentation made this present use desirable.

curves represent the results from an identical sample of wire (no. 1), which was, however, subjected to various treatments to be described later. During the experiments the load, the length of wire and the moment of inertia of the pendulum were always the same. It is seen that the maximum variation in period is about 3.85 seconds or a variation of 16 per cent. The significance of such a wide variation in the periods of torsional vibrations is very evident. It is also highly probable that the variation is too complicated to be corrected for, in any apparatus in which these suspensions are used.

The Effect of Drawing.—It was soon found that each state was quite stable in itself and unless the wires were subjected to vigorous treatment, the curves for a given sample of wire were always of a definite type. This was especially true of states II. and III. This discovery made it possible to study more carefully, each state.

When the work was undertaken, it was thought desirable to note the effect of drawing on the wires. A sample of each of the seven smallest of the wires was tested, and period-amplitude curves were plotted.

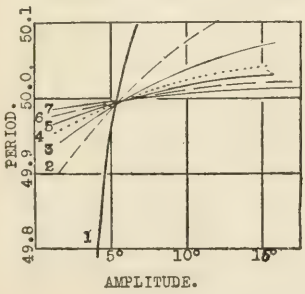

Fig. 2.

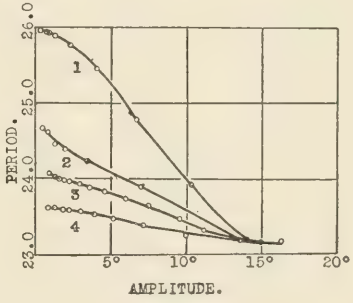

Fig. 3.

Fig. 2 shows these curves reduced to a common period at a chosen amplitude. The wires are arbitrarily numbered, in general inversely as the succession of the drawings. The diameters are given in the table below.

Wire No.	Diameter.
1	.100 mm.
2	.114
3	.133
4	.145
5	.165
6	.181
7	.206

There is seen a tendency for the effect to become more and more marked as the diameter of the wire is drawn smaller. The same effect is better shown in Fig. 3 which represents the curves for the four smallest wires

when these wires were thrown into state III. Here, as in Fig. 2, the curves have all been reduced to a common period at a certain amplitude. An explanation of this has been offered by Sieg,[1] who observed the same effect in platinum-iridium wires.

States II. and III.—The bringing about of certain states in the wires was at first very difficult, because of the apparent inconsistencies in the effects of annealing and artificial vibrations. From the experiments which need not be enumerated here, it became evident that the load supported by the wires, the slowness of the period of vibration, and the length of the suspension, had nothing to do with the state of the wire. But apparently identical treatments of any wire would cause sometimes one state and sometimes another. The wires were heated to different temperatures up to a glow-heat and changes of state occurred seemingly without regularity. The time during which the wire was heated and the rate of cooling after heating, gave no certain clue as to what state to expect.

Similarly, the impressed or artificial vibrations,[2] at the rate of about forty complete vibrations per minute, gave inconsistent changes of state when the duration of the vibration was short. It was noted, however, that if the vibrations were continued long enough, state II. would finally result. Only one exception to this has been found and that was a sample of wire no. 3, which was not changed from state III. in thirteen and one-half hours of continual vibration. The time required to change wire no. 4 from state III. to II. was twelve hours. Wire no. 1 required something over an hour of vibration for the same change. Thus the required time is quite long and depends upon the size and previous history of the wire. From the evidence now at hand, it seems safe to assume that this treatment will always cause state II. as the final state.

It was finally discovered that long continued annealing while the wire supported a small load, caused state III. This was accomplished by sending a moderate current through the wire and maintaining it for the desired time. Wire no. 4 was changed to state III. at every trial by allowing it to carry a current of 1 ampere for twelve hours while it supported a load of 25 grams. The same wire was, however, not changed from state II. to state III. by the same current flowing for 38 hours, when it supported a load of 154 grams.

On one occasion wire no. 1 was changed from state II. to III. simply

[1] L. P. Sieg, PHYS. REV., 35, p. 347, 1912.

[2] This was accomplished by arresting by means of stops, the supported pendulum without relieving the wire of its load, and, by means of gearing and a rack, imparting to the supporting-rod an angular harmonic motion that could be varied in period and amplitude between wide limits.

by allowing it to support a load of 27 grams undisturbed for four months. This process has not been repeated but it is very probable that this treatment is similar in effect to long-continued annealing.

From the data at hand we may say that, in general, (1) long-continued vibrating causes state II. and (2) long-continued annealing at a comparatively high temperature, with the wire supporting a small load, causes state III.

Summary and Conclusion.

The points to be emphasized in this preliminary report, are:

1. There are three distinct classes or states into which these wires can fall.

2. Drawing of the wires has a tendency to increase the effect of a varying period with the amplitude.

3. In general, long-continued vibrating results in a wire's falling into what I have called state II.

4. In general, long-continued annealing results in a wire's falling into state III.

In conclusion we may say that phosphor-bronze wires are not desirable as delicate suspensions, in many cases where they are used. While these peculiarities may be typical of this alloy alone, it is very probable that all alloys have their distinct peculiarities just as platinum-iridium wires and phosphor-bronze wires have been found to have. It thus seems desirable that a careful study be made of all alloys, both from the standpoint of their crystal structure with the resulting elastic constants, and from the standpoint of their practical uses.

In conclusion I wish to acknowledge my indebtedness to the staff of the department of physics for their interest in the problem, and especially to Dr. Sieg for suggesting it, and for his encouragement and assistance during the progress of the work.

Physics Laboratory,
State University of Iowa.

[Reprinted from the PHYSICAL REVIEW, N.S., Vol. VIII., No. 5, November, 1916.]

THE FREQUENT BURSTING OF HOT WATER PIPES IN HOUSEHOLD PLUMBING SYSTEMS.

BY F. C. BROWN.

IT seems to have been a common observation by plumbers that the pipes carrying the hot water from the furnace to the kitchen and bath room burst from freezing more frequently than do the pipes carrying the cold water. It is said that the ratio of frequency is at least four to one. The "cold water" usually freezes so as to lessen the flow of water in the pipes or to stop it entirely, but this freezing is seldom accompanied by bursting. If a long connection of pipe falls below the freezing temperature the probability of bursting is enhanced by virtue of its length. The bursting of the cold-water pipes is generally expected when the freezing temperature is very low. That "hot water" pipes should burst more readily than those carrying cold water might appear anomalous. Thus the experiments described in this paper were undertaken first to verify the observations of the plumbers and then to ascertain the physical explanation.

In the experiments an exact duplication of the conditions in the plumbing system were not obtained, but they were approached in some essentials by using glass test tubes closed at one end instead of iron pipe. Ordinary tap water freshly drawn was placed in these tubes to simulate the conditions in the cold-water pipes. The same tap water shortly after being boiled was placed in like tubes to simulate the conditions in the hot-water pipes. The glass tubes were satisfactory in that the visible appearances inside the tube offered valuable information as to what was happening in the respective cases. The breaking stress of the glass was materially lower than that of the ordinary iron pipe, but expected differences from this cause would merely increase the number of tubes bursting in a given time. The ordinary iron pipe that is stopped by freezing may generally be regarded as an open pipe, contrary to the glass tubes used which had one end closed. The closed pipe would have an increased probability of bursting, because of the lessened opportunity for the release of pressure by the escape of water and ice.

The glass tubes seemed to satisfactorily duplicate the conditions of the plumbing system in that the tubes containing the boiled water did

burst more frequently than those with the unboiled water. In the preliminary trials, test tubes of varying size and number were placed in the open air when the outdoor temperatures were at varying degrees below zero. The time for freezing varied from thirty minutes to four hours. The hot and cold water were placed in alternate tubes side by side. The tubes were usually about one centimeter in diameter. On seven occasions about 50 pairs of tubes were tried. In every test the tubes of boiled water broke first. When the observations were concluded it was noted that 44 tubes containing boiled water had burst and only four of those filled with unboiled water had broken. For one reason or another slightly more than half of the tubes did not break, even though all the water seemed to be frozen. Having verified the observations of the plumbers with the altered conditions, the next step was to obtain the explanation. The rate of cooling and the temperature at which freezing occurred furnished information that limited the possible explanations. In this experiment both kinds of water were allowed to reach approximate room temperatures before placement in the freezing surroundings outside.

It was found that the boiled water was invariably undercooled several degrees below zero before freezing commenced and that after crystallization began the temperature remained at zero until the entire mass was frozen. The ice was quite clear and solid. The unboiled water always began freezing at zero and the ice was full of air bubbles and appeared quite slushy, particularly near the central axis of the tube. These observations are quite in accordance with the well-known differences existing between the freezing of air-free water and water saturated with air. H. T. Barnes[1] states that the more the air and ice are mixed the less can the mixture be cooled below zero. When chilled air was passed through the water he obtained frazil or slush ice, lacking in solid structure. He further states that "with agitation, and the presence of dust suspended matter, and particularly with dissolved air in the water, uper-cooling without ice forming may be said to be impossible." Thus the observed differences between the freezing of boiled and unboiled tap water are such as can be explained by the presence or absence of dissolved air. The white spongy ice core along the central axis of the tubes in our experiments approaches in a small way the properties of frazil ice.

The fact that the temperature of the tap water during freezing did not fall below zero is indicative that the impurities, such as dissolved salts, were comparatively small. The explanation then seemed rather

[1] Ice Formation, published by Wiley, 1906.

to involve the super-cooling of the boiled water and the more solid ice formed in it. Both of these are largely dependent on the diminished amount of air in the boiled water.

The ice formed from the boiled water was clear without a decided central region often observed in artificial ice, while the ice formed from tap water was full of air bubbles especially toward the middle. These air bubbles together with accompanying impurities form nuclei for crystallization and also act as cushions to react against the expansion of the water by freezing. The air also tends to freeze out toward the middle. It will be shown that this semi-open center acts as a safety valve to relieve the pressure caused by freezing. The undercooling of the air-free water tends to make the center of the tube freeze just as solid as the water near the surface of the tube.

The occluded air also seems to make the ice more fluid in its nature. In 28 tubes side by side, which were alternately filled with boiled and unboiled water, to the same depth in each case, there was observed in every case to be a greater rise of ice in the tubes in which the unboiled water froze. Quantitative heat measurements showed that approximately the same amount of ice was formed in each case. Therefore the ice from the tap water either was less viscous than the other ice or the central core of the tap water ice remained mobile sufficiently long to relieve the pressure by carrying water and ice crystals along the tube to regions of less stress. Koch[1] found the ice containing air bubbles to have a lower elasticity than air free ice. In other words air bubbles weakens the ice. Quincke[2] observed when water containing both air and dissolved salts began to freeze that both the air and salts separated out from the ice alike, leaving the mother liquid excessively rich in both air and salts. This explains why the central core in the frozen tap water was particularly full of air bubbles. This central core either remained open or froze into a slushy ice mixture. In either case this formed an easy avenue for the release of the water pressure that tended to exist by virtue of the ice freezing along the outer wall of the tube.

The foregoing conclusion that the air in ordinary tap water is responsible for the non-bursting of cold-water pipes in many instances was further checked as follows. Water that had been boiled was afterward saturated with air by passing a stream of air through it for several minutes. Under these circumstances six pairs of tubes, alternately boiled and unboiled water, burst at about the same time. However the bursting was delayed as compared with air-free water even when the freezing temperature was quite low.

[1] Ann. d. Phys., 41, pp. 709–727, 1913.

[2] Proc. Roy. Soc. Canada, 3, p. 24, 1909.

A further experiment was made to be certain that the presence of dissolved salts was of no special importance to the observed phenomenon. Distilled water which had been exposed to the air for several days was substituted for the tap water. One of every pair of tubes was filled with this unboiled distilled water. Every alternate tube was filled with the same water after it had been reboiled. As expected the tubes containing the boiled water burst first. A final test was made by saturating this distilled boiled water with air. This procedure caused all the tubes to behave alike in freezing surroundings.

In view of these experiments it has been concluded that the occluded air in ordinary tap water is responsible for the delay or absence of bursting of the pipes. The air and accompanying impurities assist in furnishing nuclei of crystallization, so that the ordinary tap water begins to freeze at zero degree. At the same time the ice formed is more mobile, especially near the middle of the tube, so that until very low temperatures are reached the pressure is released along the middle of the tube by the flow of water and ice. In addition the air bubbles displacing water form cushions which relieve the pressure on the tube to a certain extent.

The writer is indebted to Mr. Waldemar Noll for very able assistance, particularly in connection with the experimental work here described.

IN THE SERIES ON AIMS AND PROGRESS OF RESEARCH

TABLE OF LOG_{10} SEC^2 θ FOR DETERMINING PHOTOGRAPHIC DENSITIES BY MEANS OF NICOL PRISMS

COMPUTED BY PAUL S. HELMICK

COPIES MAY BE OBTAINED ON REQUEST
FROM THE LIBRARIAN OF THE UNIVERSITY

PUBLICATIONS IN THIS SERIES

1	*Child welfare research station*	Jan. 15, 1916
2	*Vocational guidance in music*	Sept. 1916
3	*Schedule of dissertations*	Mar. 1917
4	*Table of* $\log_{10} \sec^2\theta$ *for determining photographic densities by means of nicol prisms*	May, 1917

TABLE OF LOG_{10} $\text{SEC}^2\,\theta$ FOR DETERMINING PHOTOGRAPHIC DENSITIES BY MEANS OF NICOL PRISMS

COMPUTED BY PAUL S. HELMICK

The following table was computed for use with photographic density determining apparatus employing nicol prisms in their construction. Nutting's photometric attachment*, Wallace's modification of Brace's prism photometer†, and Lemon's form of a polarization spectrophotometer§, are all instruments which can be successfully employed in photographic density measurements, and all make use of nicol prisms.

The photographic density corresponding to an angle of θ degrees between transmitting planes of the nicols when rotated so as to cut down one light beam to the same intensity which the interposed plate of unknown density cuts down the other light beam, is given by the expression $\text{Log}_{10}\ \text{Sec}^2\theta$. Lemon‖ gives a four-place table of $\sin^2\theta$ in degrees and tenths, from which, with the additional aid of a table of logarithms and photographic density corresponding to an angle of rotation of the nicols may be found.

In his work, the writer has felt keenly the need of a more extensive table giving density directly in terms of the angle of rotation of the nicols, and has therefore computed the following table of $\text{Log}_{10}\ \text{Sec}^2\theta$, and has appended values of differences for 0.01 degree. It is his sincere hope that the table will be of service to others working in the field of photographic density determinations.

**Bulletin of the Bureau of Standards*, 7 (1911), p. 239
†*Astrophysical Journal*, 25 (1911), p. 124.
§*Astrophysical Journal*, 39 (1914), p. 204.
‖*loc. cit.*

	.00	.05	.10	.15	.20	.25	.30	.35	.40	.45
0	0.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00002	.00002	.00002
	0.0	0.0	0.0	0.0	0.0	0.0	0.4	0.0	0.0	0.0
1	0.00014	.00014	.00016	.00018	.00020	.00020	.00022	.00024	.00026	.00028
	0.0	0.4	0.4	0.4	0.0	0.4	0.4	0.4	0.4	0.4
2	0.00052	.00056	.00058	.00062	.00064	.00066	.00070	.00074	.00076	.00080
	0.8	0.4	0.8	0.4	0.4	0.8	0.8	0.4	0.8	0.4
3	0.00120	.00124	.00128	.00132	.00136	.00140	.00144	.00148	.00154	.00158
	0.8	0.8	0.8	0.8	0.8	0.8	0.8	1.2	0.8	0.8
4	0.00212	.00218	.00222	.00228	.00234	.00240	.00244	.00250	.00256	.00262
	1.2	0.8	1.2	1.2	1.2	0.8	1.2	1.2	1.2	1.2
5	0.00332	.00338	.00344	.00352	.00358	.00366	.00372	.00380	.00386	.00394
	1.2	1.2	1.6	1.2	1.6	1.2	1.6	1.2	1.6	1.2
6	0.00478	.00486	.00494	.00502	.00510	.00518	.00526	.00534	.00544	.00552
	1.6	1.6	1.6	1.6	1.6	1.6	1.6	2.0	1.6	1.6
7	0.00650	.00660	.00668	.00678	.00688	.00698	.00706	.00716	.00726	.00736
	2.0	1.6	2.0	2.0	2.0	1.6	2.0	2.0	2.0	2.0
8	0.00850	.00860	.00870	.00882	.00892	.00904	.00914	.00926	.00936	.00948
	2.0	2.0	2.4	2.0	2.4	2.0	2.4	2.0	2.4	2.4
9	0.01076	.01088	.01100	.01112	.01124	.01136	.01150	.01162	.01174	.01186
	2.4	2.4	2.4	2.4	2.4	2.8	2.4	2.4	2.4	2.8
10	0.01330	.01344	.01356	.01370	.01384	.01398	.01412	.01424	.01438	.01452
	2.8	2.4	2.8	2.8	2.8	2.8	2.4	2.8	2.8	2.8
11	0.01610	.01626	.01640	.01256	.01670	.01686	.01700	.01716	.01730	.01746
	3.2	2.8	3.2	2.8	3.2	2.8	3.2	2.8	3.2	3.2
12	0.01920	.01936	.01952	.01968	.01984	.02000	.02018	.02034	.02050	.02066
	3.2	3.2	3.2	3.2	3.2	3.6	3.2	3.2	3.2	3.2
13	0.02256	.52272	.02290	.03208	.02326	.02344	.02362	.02380	.02398	.02416
	3.2	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6
14	0.02620	.02638	.02658	.02676	.02696	.02714	.02734	.02754	.02772	.02792
	3.6	4.0	3.6	4.0	3.6	4.0	4.0	3.6	4.0	4.0
15	0.03012	.03032	.03052	.03072	.03094	.03114	.03134	.03156	.03176	.03196
	4.0	4.0	4.0	4.4	4.0	4.0	4.4	4.0	4.0	4.0

	.50	.55	.60	.65	.70	.75	.80	.85	.90	.95
0	0.00004	.00004	.00004	.00006	.00006	.00008	.00008	.00010	.00010	.00012
	0.0	0.0	0.4	0.0	0.4	0.0	0.4	0.0	0.4	0.4
1	0.00030	.00032	.00034	.00036	.00038	.00040	.00042	.00046	.00048	.00050
	0.4	0.4	0.4	0.4	0.4	0.4	0.8	0.4	0.4	0.4
2	0.00082	.00086	.00090	.00092	.00096	.00100	.00104	.00108	.00112	.00116
	0.8	0.8	0.4	0.8	0.8	0.8	0.8	0.8	0.8	0.8
3	0.00162	.00166	.00172	.00176	.00182	.00186	.00192	.00196	.00202	.00206
	0.8	1.2	0.8	1.2	0.8	1.2	0.8	1.2	0.8	1.2
4	0.00268	.00274	.00280	.00286	.00292	.00298	.00306	.00312	.00318	.00324
	1.2	1.2	1.2	1.2	1.2	1.6	1.2	1.2	1.2	1.6
5	0.00400	.00408	.00416	.00424	.00430	.00438	.00446	.00454	.00462	.00470
	1.6	1.6	1.6	1.2	1.6	1.6	1.6	1.6	1.6	1.6
6	0.00560	.00568	.00578	.00586	.00596	.00604	.00614	.00622	.00632	.00640
	1.6	2.0	1.6	2.0	1.6	2.0	1.6	2.0	1.6	2.0
7	0.00746	.00756	.00766	.00776	.00786	.00798	.00808	.00818	.00828	.00838
	2.0	2.0	2.0	2.0	2.4	2.0	2.0	2.0	2.0	2.4
8	0.00960	.00970	.00982	.00994	.01006	.01016	.01028	.01040	.01052	.01063
	2.0	2.4	2.4	2.4	2.0	2.4	2.4	2.4	2.4	2.4
9	0.01200	.01212	.01224	.01238	.01250	.01264	.01276	.01290	.01304	.01316
	2.4	2.4	2.8	2.4	2.8	2.4	2.8	2.8	2.4	2.8
10	0.01466	.01480	.01496	.01510	.01524	.01538	.01552	.01566	.01582	.01596
	2.8	3.2	2.8	2.8	2.8	2.8	2.8	3.2	2.8	2.8
11	0.01762	.01776	.01782	.01808	.01824	.01840	.01856	.01872	.01888	.01904
	2.8	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2
12	0.02082	.02100	.02118	.02134	.02152	.02168	.02186	.02204	.02220	.02238
	3.6	3.6	3.2	3.6	3.2	3.6	3.6	3.2	3.6	3.6
13	0.02434	.02452	.02470	.02488	.02508	.02526	.02544	.02562	.02582	.02600
	3.6	3.6	3.6	4.0	3.6	3.6	3.6	4.0	3.6	4.0
14	0.02812	.02832	.02852	.02870	.02890	.02910	.02930	.02950	.02970	.02990
	4.0	4.0	3.6	4.0	4.0	4.0	4.0	4.0	4.0	4.4
15	0.03218	.03238	.03260	.03282	.03302	.03324	.03346	.03366	.03388	.03410
	4.0	4.4	4.4	4.0	4.4	4.4	4.0	4.4	4.4	4.4

	.00	.05	.10	.15	.20	.25	.30	.35	.40	.45
15	0.03012	.03032	.03052	.03072	.03094	.03114	.03134	.03156	.03176	.03196
	4.0	4.0	4.0	4.4	4.0	4.0	4.4	4.0	4.0	4.0
16	0.03432	.03454	.03476	.03498	.03520	.03542	.03564	.03586	.03608	.03630
	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4
17	0.03880	.03904	.03928	.03950	.03974	.03998	.04022	.04044	.04068	.04092
	4.8	4.8	4.4	4.8	4.8	4.8	4.4	4.8	4.8	4.8
18	0.04358	.04384	.04408	.04432	.04458	.04482	.04508	.04532	.04558	.04584
	5.2	4.8	4.8	5.2	4.8	5.2	4.8	5.2	5.2	4.8
19	0.04866	.04892	.04918	.04944	.04970	.04998	.05024	.05050	.05078	.05104
	5.2	5.2	5.2	5.2	5.6	5.2	5.2	5.6	5.2	5.2
20	0.05402	.05430	.05458	.05486	.05514	.05542	.05570	.05598	.05626	.05654
	5.6	5.6	5.6	5.6	5.6	5.6	5.6	5.6	5.6	5.6
21	0.05970	.05998	.06028	.06058	.06086	.06116	.06146	.06176	.06204	.06234
	5.6	6.0	6.0	5.6	6.0	6.0	6.0	5.6	6.0	6.0
22	0.06566	.06598	.06628	.06660	.06690	.06720	.06752	.06784	.06814	.06846
	6.4	6.0	6.4	6.0	6.0	6.4	6.4	6.0	6.4	6.0
23	0.07194	.07226	.07260	.07292	.07324	.07356	.07390	.07422	.07454	.07488
	6.4	6.8	6.4	6.4	6.4	6.8	6.4	6.4	6.8	6.4
24	0.07854	.07888	.07922	.07956	.07990	.08024	.08058	.08092	.08126	.08160
	6.8	6.8	6.8	6.8	6.8	6.8	6.8	6.8	6.8	7.2
25	0.08544	.08580	.08616	.08652	.08686	.08722	.08758	.08794	.08830	.08866
	7.2	7.2	7.2	6.8	7.2	7.2	7.2	7.2	7.2	7.2
26	0.09268	.09304	.09342	.09380	.09416	.09454	.09492	.09528	.09566	.09604
	7.2	7.6	7.6	7.2	7.6	7.6	7.2	7.6	7.6	7.6
27	0.10024	.10062	.10102	.10140	.10178	.10218	.10258	.10296	.10336	.10374
	7.6	8.0	7.6	7.6	8.0	8.0	7.6	8.0	7.6	8.0
28	0.10814	.10854	.10894	.10934	.10974	.11016	.11056	.11098	.11138	.11180
	8.0	8.0	8.0	8.0	8.4	8.0	8.4	8.0	8.4	8.0
29	0.11636	.11678	.11720	.11762	.11804	.11848	.11890	.11932	.11976	.12018
	8.4	8.4	8.4	8.4	8.8	8.4	8.4	8.8	8.4	8.4
30	0.12494	.12538	.12582	.12626	.12670	.12714	.12758	.12802	.12846	.12892
	8.8	8.8	8.8	8.8	8.8	8.8	8.8	8.8	9.2	8.8

	.50	.55	.60	.65	.70	.75	.80	.85	.90	.95
15	0.03218	.03238	.03260	.03282	.03302	.03324	.03346	.03366	.03388	.03410
	4.0	4.4	4.4	4.0	4.4	4.4	4.4	4.4	4.4	4.4
16	0.03652	.03676	.03698	.03720	.03740	.03766	.03788	.03812	.03834	.03858
	4.8	4.4	4.4	4.4	4.8	4.4	4.8	4.4	4.8	4.4
17	0.04116	.04140	.04164	.04188	.04212	.04236	.04260	.04286	.04310	.04334
	4.8	4.8	4.8	4.8	4.8	4.8	5.2	5.2	4.8	4.8
18	0.04608	.04634	.04660	.04686	.04710	.04736	.04762	.04788	.04814	.04840
	5.2	5.2	5.2	4.8	5.2	5.2	5.2	5.2	5.2	5.2
19	0.05130	.05158	.05184	.05212	.05238	.05266	.05294	.05320	.05348	.05376
	5.6	5.2	5.6	5.2	5.6	5.6	5.2	5.6	5.6	5.2
20	0.05682	.05710	.05740	.05768	.05796	.05826	.05854	.05882	.05912	.05940
	5.6	6.0	5.6	5.6	6.0	5.6	5.6	6.0	5.6	6.0
21	0.06264	.06294	.06324	.06354	.06384	.06414	.06444	.06476	.06506	.06536
	6.0	6.0	6.0	6.0	6.0	6.0	6.4	6.0	6.0	6.0
22	0.06876	.06908	.06940	.06972	.07004	.07034	.07066	.07098	.07130	.07162
	6.4	6.4	6.4	6.4	6.0	6.4	6.4	6.4	6.4	6.4
23	0.07520	.07554	.07586	.07620	.07652	.07686	.07720	.07754	.07786	.07820
	6.8	6.4	6.8	6.4	6.8	6.8	6.8	6.4	6.8	6.8
24	0.08196	.08230	.08264	.08300	.08334	.08370	.08404	.08440	.08474	.08510
	6.8	6.8	7.2	6.8	7.2	6.8	7.2	6.8	7.2	6.8
25	0.08902	.08938	.08974	.09012	.09048	.09084	.09120	.09158	.09194	.09232
	7.2	7.2	7.6	7.2	7.2	7.2	7.6	7.2	7.6	7.2
26	0.09642	.09680	.09718	.09756	.09794	.09830	.09870	.09909	.09946	.09986
	7.6	7.6	7.6	7.6	7.6	7.6	7.6	7.6	8.0	7.6
27	0.10414	.10454	.10494	.10532	.10572	.10612	.10652	.10692	.10732	.10772
	8.0	8.0	7.6	8.0	8.0	8.0	8.0	8.0	8.0	8.4
28	0.11220	.11262	.11302	.11344	.11386	.11428	.11468	.11510	.11552	.11594
	8.4	8.0	8.4	8.4	8.4	8.0	8.4	8.4	8.4	8.4
29	0.12060	.12104	.12146	.12190	.12232	.12276	.12320	.12362	.12406	.12450
	8.8	8.4	8.8	8.4	8.8	8.8	8.4	8.8	8.8	8.8
30	0.12936	.12980	.13026	.13070	.13116	.13160	.13206	.13250	.13296	.13342
	8.8	9.2	8.8	9.2	8.8	9.2	8.8	9.2	9.2	8.8

	.00	.05	.10	.15	.20	.25	.30	.35	.40	.45
30	0.12494	.12538	.12582	.12626	.12670	.12714	.12758	.12802	.12846	.12892
	8.8	8.8	8.8	8.8	8.8	8.8	8.8	8.8	9.2	8.8
31	0.13386	.13432	.13478	.13524	.13570	.13616	.13662	.13708	.13754	.13800
	9.2	9.2	9.2	9.2	9.2	9.2	9.2	9.2	9.2	9.2
32	0.14316	.14364	.14410	.14458	.14506	.14554	.14602	.14650	.14698	.14746
	9.6	9.2	9.6	9.6	9.6	9.6	9.6	9.6	9.6	9.6
33	0.15282	.15330	.15380	.15430	.15480	.15530	.15578	.15628	.15678	.15728
	9.6	10	10	10	10	9.6	10	10	10	10
34	0.16286	.16336	.16388	.16438	.16490	.16542	.16594	.16646	.16698	.16750
	10	10	10	10	10	10	10	10	10	10
35	0.17328	.17380	.17434	.17486	.17540	.17594	.17648	.17702	.17754	.17808
	10	11	10	11	11	11	11	10	11	11
36	0.18408	.18464	.18518	.18574	.18630	.18686	.18740	.18796	.18852	.18908
	11	11	11	11	11	11	11	11	11	11
37	0.19530	.19588	.19644	.19702	.19760	.19818	.19874	.19932	.19990	.20048
	12	11	12	12	12	11	12	12	12	12
38	0.20694	.20752	.20812	.20872	.20932	.20992	.21050	.21110	.21170	.21230
	12	12	12	12	12	12	12	12	12	12
39	0.21900	.21960	.22022	.22084	.22146	.22208	.22270	.22332	.22394	.22456
	12	12	12	12	12	12	12	12	12	12
40	0.23150	.23212	.23276	.23340	.23404	.23468	.23532	.23598	.23662	.23726
	12	13	13	13	13	13	13	13	13	13
41	0.24444	.24510	.24576	.24642	.24708	.24774	.24842	.20908	.24974	.25042
	13	13	13	13	13	14	13	13	14	13
42	0.25786	.25854	.25922	.25990	.26060	.26128	.26196	.26266	.26336	.26404
	14	14	14	14	14	14	14	14	14	14
43	0.27140	.27246	.27316	.27388	.27458	.27530	.27600	.27672	.27744	.27816
	14	14	14	14	14	14	14	14	14	14
44	0.28614	.28686	.28760	.28834	.28906	.28980	.29054	.29128	.29202	.29278
	14	15	15	14	15	15	15	15	15	15
45	0.30103	.30178	.30254	.30330	.30408	.30484	.30560	.30636	.30714	.30790
	15	15	15	16	15	15	15	16	15	16

	.50	.55	.60	.65	.70	.75	.80	.85	.90	.95
30	0.12936	.12980	.13026	.13070	.13116	.13160	.13206	.13250	.13296	.13342
	8.8	9.2	8.8	9.2	8.8	9.2	8.8	9.2	9.2	8.8
31	0.13846	.13894	.13940	.13986	.14034	.14080	.14128	.14174	.14222	.14268
	9.6	9.2	9.2	9.6	9.2	9.6	9.2	9.6	9.2	9.6
32	0.14794	.14842	.14890	.14940	.14988	.15036	.15086	.15134	.15184	.15232
	9.6	9.6	10	9.6	9.6	10	9.6	10	9.6	10
33	0.15778	.15828	.15880	.15930	.15980	.16030	.16082	.16132	.16184	.16234
	10	10	10	10	10	10	10	10	10	10
34	0.16802	.16854	.16906	.16958	.17010	.17062	.17116	.17168	.17222	.17274
	10	10	10	10	10	11	10	11	10	11
35	0.17862	.17916	.17972	.18026	.18080	.18134	.18188	.18244	.18298	.18354
	11	11	11	11	11	11	11	11	11	11
36	0.18964	.19020	.19076	.19132	.19190	.19246	.19302	.19360	.19416	.19474
	11	11	11	12	11	11	12	11	12	11
37	0.20106	.20164	.20224	.20282	.20340	.20398	.20458	.20516	.20576	.20634
	12	12	12	12	12	12	12	12	12	12
38	0.21292	.21352	.21412	.21472	.21534	.21594	.21654	.21716	.21776	.21838
	12	12	12	12	12	12	12	12	12	12
39	0.22518	.22582	.22644	.22706	.22770	.22832	.22986	.22958	.23022	.23086
	13	12	12	13	12	13	12	13	13	13
40	0.23790	.23856	.23920	.23986	.24050	.24116	.24182	.24246	.24312	.24378
	13	13	13	13	13	13	13	13	13	13
41	0.25108	.25176	.25244	.25310	.25378	.25446	.25514	.25582	.25650	.25718
	14	14	13	14	14	14	14	14	14	14
42	0.26474	.26544	.26612	.26682	.26752	.26822	.26892	.26964	.27034	.27104
	14	14	14	14	14	14	14	14	14	14
43	0.27888	.27960	.28032	.28104	.28176	.28248	.28322	.28394	.28468	.28540
	14	14	14	14	14	15	14	15	14	15
44	0.29352	.29426	.29500	.29576	.29650	.29726	.29800	.29876	.29952	.30028
	15	15	15	15	15	15	15	15	15	15
45	0.30868	.30944	.31022	.31100	.31178	.31254	.31332	.31410	.31490	.31568
	15	16	16	16	15	16	16	16	16	16

	.00	.05	.10	.15	.20	.25	.30	.35	.40	.45
45	0.30103	.30178	.30254	.30330	.30408	.30484	.30560	.30636	.30714	.30790
	15	15	15	16	15	15	15	16	15	16
46	0.31646	.31724	.31804	.31882	.31960	.32040	.32120	.32198	.32278	.32358
	16	16	16	16	16	16	16	16	16	16
47	0.33244	.33324	.33406	.33488	.33570	.33652	.33734	.33816	.33898	.33980
	16	16	16	16	16	16	16	16	16	17
48	0.34898	.34982	.35066	.35152	.35236	.35320	.35406	.35490	.35576	.35662
	17	17	17	17	17	17	17	17	17	17
49	0.36612	.36698	.36786	.36874	.36962	.37050	.37138	.37226	.37314	.37402
	17	18	18	18	18	18	18	18	18	18
50	0.38386	.38476	.38568	.38658	.38750	.38840	.38932	.39022	.39114	.39206
	18	18	18	18	18	18	18	18	18	18
51	0.40226	.40320	.40414	.40508	.40602	.40696	.40790	.40884	.40980	.41074
	19	19	19	19	19	19	19	19	19	19
52	0.42132	.42228	.42326	.42424	.42522	.42618	.42716	.42816	.42914	.43012
	19	20	20	20	19	20	20	20	20	20
53	0.44108	.44208	.44308	.44410	.44512	.44612	.44714	.44816	.44918	.45020
	20	20	20	20	20	20	20	20	20	20
54	0.46156	.46260	.46366	.46470	.46576	.46680	.46786	.46892	.46998	.47104
	21	21	21	21	21	21	21	21	21	21
55	0.48282	.48390	.48598	.48608	.48716	.48826	.48934	.49044	.49154	.49264
	22	22	22	22	22	22	22	22	22	22
56	0.50488	.50600	.50712	.50826	.50938	.51052	.51166	.51280	.51394	.51508
	22	22	23	22	23	23	23	23	23	23
57	0.52778	.52896	.53012	.53130	.53246	.53364	.53482	.53600	.53720	.53838
	24	23	24	23	24	24	24	24	24	24
58	0.55158	.55280	.55402	.55524	.55646	.55768	.55990	.56012	.56136	.56260
	24	24	24	24	24	24	24	25	25	24
59	0.57632	.57758	.57884	.58012	.58138	.58266	.58394	.58522	.58650	.58778
	25	25	26	25	26	26	26	26	26	26
60	0.60206	.60338	.60470	.60602	.60734	.60866	.60998	.61132	.61264	.61398
	26	26	26	26	26	26	27	26	27	27

	.50	.55	.60	.65	.70	.75	.80	.85	.90	.95
45	0.30868	.30944	.31022	.31100	.31178	.31254	.31332	.31410	.31490	.31568
	15	16	16	16	15	16	16	16	16	16
46	0.32438	.32518	.32598	.32678	.32758	.32838	.32920	.33000	.33082	.33162
	16	16	16	16	16	16	16	16	16	16
47	0.34064	.34146	.34230	.34312	.34396	.34478	.34562	.34646	.34730	.34814
	16	17	16	17	16	17	17	17	17	17
48	0.35748	.35832	.35918	.36004	.36090	.36178	.36264	.36350	.36438	.36524
	17	17	17	17	18	17	17	18	17	18
49	0.37492	.37580	.37668	.37758	.37848	.37936	.38026	.38116	.38206	.38296
	18	18	18	18	18	18	18	18	18	18
50	0.39298	.39390	.39482	.39574	.39668	.39766	.39852	.39946	.40038	.40132
	18	18	18	19	18	18	19	18	19	19
51	0.41170	.41266	.41362	.41456	.41552	.41648	.41744	.41842	.41938	.42034
	19	19	19	19	19	19	20	19	19	19
52	0.43110	.43210	.43308	.43408	.43508	.43606	.43706	.43806	.43906	.44006
	20	20	20	20	20	20	20	20	20	20
53	0.45122	.45226	.45328	.45430	.45534	.45638	.45740	.45844	.45948	.46052
	21	20	20	21	21	20	21	21	21	21
54	0.47210	.47316	.47422	.47528	.47636	.47742	.47850	.47958	.48066	.48174
	21	21	21	22	21	22	22	22	22	22
55	0.49374	.49484	.49596	.49706	.49818	.49928	.50040	.50152	.50264	.50376
	22	22	22	22	22	22	22	22	22	22
56	0.51622	.51736	.51852	.51966	.52082	.52198	.52314	.52430	.52546	.52662
	23	23	23	23	23	23	23	23	23	23
57	0.53956	.54076	.54196	.54314	.54434	.54554	.54674	.54796	.54916	.55036
	24	24	24	24	24	24	24	24	24	24
58	0.56382	.56506	.56630	.56756	.56880	.57004	.57130	.57254	.57380	.57506
	25	25	25	25	25	25	25	25	25	25
59	0.58906	.59036	.59164	.59294	.59424	.59522	.59682	.59814	.59944	.60074
	26	26	26	26	26	26	26	26	26	26
60	0.61532	.61666	.61800	.61936	.62079	.62206	.62342	.62476	.62612	.62750
	27	27	27	27	27	27	27	27	28	27

	.00	.05	.10	.15	.20	.25	.30	.35	.40	.45
60	0.60206	.60338	.60470	.60602	.60734	.60866	.60998	.61132	.61264	.61398
	26	26	26	26	26	26	27	26	27	27
61	0.62886	.63022	.63160	.63298	.63434	.63574	.63712	.63850	.63988	.64128
	27	28	28	27	28	28	28	28	28	28
62	0.65678	.65820	.65964	.66108	.66250	.66394	.66538	.66684	.66828	.66974
	28	29	29	28	29	29	29	29	29	29
63	0.68590	.68740	.68888	.69038	.69188	.69338	.69490	.69640	.69792	.69942
	30	30	30	30	30	30	30	30	30	30
64	0.71632	.71788	.71944	.72100	.72256	.72412	.72570	.72728	.72886	.73044
	31	31	31	31	31	32	32	32	32	32
65	0.74810	.74974	.75136	.75300	.75464	.75628	.75792	.75958	.76122	.76288
	33	32	33	33	33	33	33	33	33	33
66	0.78138	.78308	.78478	.78650	.78822	.78994	.79166	.79338	.79512	.79680
	34	34	34	34	34	34	34	35	35	35
67	0.81624	.81804	.81982	.82162	.82342	.82522	.82704	.82886	.83066	.83250
	36	36	36	36	36	36	36	36	37	36
68	0.85284	.85472	.85662	.85850	.86040	.86228	.86420	.86610	.86802	.86992
	38	38	38	38	38	38	38	38	38	38
69	0.89134	.89332	.89530	.89728	.89928	.90128	.90328	.90530	.90730	.90932
	40	40	40	40	40	40	40	40	40	40
70	0.93190	.93398	.93608	.93816	.94028	.94238	.94450	.94662	.94874	.95088
	42	42	42	42	42	42	42	42	43	42
71	0.97472	.97692	.97914	.98134	.98358	.98580	.98804	.99028	.99252	.99478
	44	44	44	45	44	45	45	45	45	45
72	1.02004	.02238	.02472	.02706	.02942	.03178	.03416	.03654	.03892	.04132
	47	47	47	47	47	48	48	48	48	48
73	1.06812	.07062	.07310	.07560	.07810	.08062	.08314	.08568	.08822	.09076
	50	50	50	50	50	50	51	51	51	51
74	1.11932	.12198	.12462	.12730	.12996	.13266	.13534	.13804	.14076	.14348
	53	53	54	53	54	54	54	54	54	54
75	1.17400	.17684	.17968	.18254	.18540	.18828	.19116	.19406	.19696	.19988
	57	57	57	57	58	58	58	58	58	58

	.50	.55	.60	.65	.70	.75	.80	.85	.90	.95
60	0.61532	.61666	.61800	.61936	.62070	.62206	.62342	.62476	.62612	.62750
	27	27	27	27	27	27	27	27	28	27
61	0.64268	.64408	.64548	.64688	.64828	.64970	.65112	.65252	.65394	.65536
	28	28	28	28	28	28	28	28	28	28
62	0.67118	.67264	.67410	.67558	.67704	.67850	.67998	.68146	.68294	.68442
	29	29	30	29	29	30	30	30	30	30
63	0.70094	.70246	.70400	.70552	.70706	.70858	.71012	.71166	.71322	.71476
	30	31	30	31	30	31	31	31	31	31
64	0.73204	.73662	.73522	.73682	.73842	.74002	.74164	.74324	.74486	.74648
	32	32	32	32	32	32	32	32	32	32
65	0.76454	.76622	.76788	.76956	.77124	.77292	.77460	.77628	.77798	.77968
	34	33	34	34	34	34	34	34	34	34
66	0.79860	.80034	.80210	.80384	.80560	.80736	.80914	.81090	.81268	.81446
	35	35	35	35	35	36	35	36	36	36
67	0.83432	.83616	.83798	.83982	.84168	.84352	.84538	.84724	.84910	.85098
	37	36	37	37	37	37	37	37	38	37
68	0.87184	.87378	.87570	.87764	.87958	.88154	.88348	.88544	.88740	.88936
	39	38	39	39	39	39	39	39	39	40
69	0.91134	.91338	.91542	.91746	.91950	.92156	.92362	.92568	.92774	.92982
	41	41	41	41	41	41	41	41	42	42
70	0.95300	.95516	.95730	.95946	.96162	.96378	.96596	.96814	.97032	.97252
	43	43	43	43	43	44	44	44	44	44
71	0.99704	.99932	*.00160	*.00388	*.00616	*.00846	*.01076	*.01306	*.01538	*.01770
	46	46	46	46	46	46	46	46	46	47
72	1.04372	.04612	.04854	.05096	.05340	.05582	.05828	.06072	.06318	.06566
	48	48	48	49	48	49	49	49	50	49
73	1.09332	.09588	.09846	.10104	.10362	.10622	.10882	.11144	.11406	.11668
	51	52	52	52	52	52	52	52	52	53
74	1.14620	.14894	.15168	.15444	.15720	.15998	.16278	.16556	.16836	.17118
	55	55	55	55	56	56	56	56	56	56
75	1.20280	.20574	.20868	.21164	.21460	.21758	.22058	.22358	.22660	.22962
	59	59	59	59	60	60	60	60	60	60

	.00	.05	.10	.15	.20	.25	.30	.35	.40	.45
75	1.17400	.17684	.17968	.18254	.18540	.18828	.19116	.19406	.19696	.19988
	57	57	57	57	58	58	58	58	58	58
76	1.23264	.23570	.23876	.24182	.24490	.24800	.25110	.25422	.25734	.26048
	61	61	61	62	62	62	62	62	63	63
77	1.29582	.29912	.30242	.30574	.30906	.31240	.31576	.31914	.32252	.32592
	66	66	66	66	67	67	68	68	68	68
78	1.36424	.36782	.37140	.37500	.37864	.38226	.38592	.38958	.39328	.39698
	72	72	72	73	72	73	73	74	74	74
79	1.43880	.44272	.44664	.45058	.45454	.45854	.46254	.46656	.47060	.47466
	78	78	79	79	80	80	80	81	81	82
80	1.52066	.52496	.52930	.53366	.53804	.54244	.54686	.55130	.55578	.56026
	86	87	87	88	88	88	89	90	90	90
81	1.61134	.61614	.62096	.62582	.63070	.63560	.64054	.64552	.65052	.65554
	96	96	97	98	98	99	100	100	100	101
82	1.71288	.71830	.72374	.72922	.73474	.74030	.74588	.75150	.75716	.76286
	108	109	110	110	111	112	112	113	114	115
83	1.82822	.83440	.84064	.84694	.85326	.85964	.86608	.87256	.87908	.88566
	124	125	126	126	128	129	130	130	132	132
84	1.96154	.96878	.97608	.98344	.99088	.99836	*.00592	*.01356	*.02126	*.02902
	145	146	147	149	150	151	153	154	155	157
85	2.11940	.12812	.13692	.14580	.15478	.16386	.17302	.18230	.19166	.20114
	174	176	178	180	182	183	186	187	190	192
86	2.31284	.32374	.33478	.34598	.35732	.36880	.38044	.39224	.40422	.41634
	218	221	224	227	230	233	236	240	242	246
87	2.56240	.57698	.59182	.60692	.62228	.63792	.65384	.67006	.68660	.70346
	292	297	302	307	313	318	324	331	337	344
88	2.91436	.93634	.95890	.98206	*.00584	*.03030	*.05548	*.08140	*.10812	*.13568
	440	451	463	476	489	540	518	534	551	570
89	3.51629	.56084	.60780	.65744	.71010	.76614	.82608	.89044	.95996	*.03554
	891	939	993	1053	1121	1199	1287	1390	1512	1656

	.50	.55	.60	.65	.70	.75	.80	.85	.90	.95
75	1.20280	.20574	.20868	.21164	.21460	.21758	.22058	.22358	.22660	.22962
	59	59	59	59	60	60	60	60	60	60
76	1.26362	.26680	.26996	.27316	.27636	.27956	.28280	.28604	.28928	.29254
	64	63	64	64	64	65	65	65	65	66
77	1.32932	.33276	.33620	.33964	.34312	.34660	.35010	.35362	.35714	.36068
	69	69	69	70	70	70	70	70	71	71
78	1.40068	.40442	.40818	.41194	.41572	.41952	.42334	.42718	.43104	.43492
	75	75	75	76	76	76	77	77	78	78
79	1.47874	.48284	.48696	.49110	.49526	.49944	.50364	.50786	.51210	.51638
	82	82	83	83	84	84	84	85	86	86
80	1.56478	.56932	.57388	.57848	.58310	.58774	.59240	.59710	.60182	.60656
	91	91	92	92	93	93	94	94	95	96
81	1.66060	.66568	.67080	.67594	.68112	.68634	.69158	.69686	.70218	.70752
	102	102	103	104	104	105	106	106	107	107
82	1.76860	.77438	.78020	.78606	.79196	.79788	.80386	.80988	.81596	.82206
	116	116	117	118	118	119	120	122	122	123
83	1.89228	.89896	.90570	.91248	.91932	.92620	.93316	.94016	.94722	.95434
	134	135	136	137	138	139	140	141	142	144
84	2.03686	.04476	.05274	.06080	.06894	.07714	.08544	.09380	.10226	.11078
	158	160	161	163	164	166	167	169	170	172
85	2.21072	.22040	.23020	.24010	.25012	.26026	.27052	.28090	.29142	.30206
	194	196	198	200	203	205	208	210	213	216
86	2.42864	.44114	.45380	.46666	.47970	.49294	.50640	.52006	.53394	.54806
	250	253	257	261	265	269	273	278	282	287
87	2.72064	.73818	.75608	.77436	.79302	.81210	.83162	.85158	.87200	.89292
	351	358	366	373	382	390	399	408	418	429
88	3.16416	.19360	.22408	.25566	.28844	.32250	.35794	.39490	.43352	.47392
	589	610	632	656	681	709	739	772	808	847
89	4.11832	.20982	.31214	.42812	.56200	.72036	.91418	*.16407	*.51624	.11831
	1830	2046	2320	2678	3167	3876	4998	7044	12041	

89°.00	3.51629	864	89°.35	.89044	1326	89°.70	.56200	2848
.01	.52502	873	.36	.90390	1346	.71	.59146	2946
.02	.53384	882	.37	.91758	1368	.72	.62194	3048
.03	.54274	890	.38	.93148	1390	.73	.65352	3158
.04	.55174	900	.39	.94560	1412	.74	.68630	3278
.05	.06084	910	.40	.95996	1436	.775	4.72036	3406
.06	.57003	919	.41	.97456	1460	.76	.75582	3546
.07	.57932	929	.42	.98940	1484	.77	.79279	3697
.08	.58870	938	.43	4.00450	1510	.78	.83140	3861
.09	.59820	950	.44	.01988	1538	.79	.87181	4041
.10	.60780	960	.45	.03554	1566	.80	.91418	4237
.11	.61750	970	.46	.05148	1594	.81	.95874	4456
.12	.62732	982	.47	.06770	1622	.82	5.00570	4696
.13	.63724	992	.48	.08426	1656	.83	.05535	4965
.14	.64728	1004	.49	.10112	1686	.84	.10801	5266
.15	.65744	1016	.50	4.11832	1720	.85	.16407	5606
.16	.66772	1028	.51	.13586	1754	.86	.22399	5992
.17	.67812	1040	.52	.15378	1792	.87	.28836	6437
.18	.68864	1052	.53	.17206	1828	.88	.35788	6952
.19	.69930	1066	.54	.19074	1868	.89	.43346	7558
.20	.71010	1080	.55	.20982	1908	.90	.51624	8278
.21	.72108	1092	.56	.22934	1952	.91	.60776	9152
.22	.73208	1106	.57	.24932	1998	.92	.71006	10230
.23	.74328	1120	.58	.26976	2044	.93	.82605	11599
.24	.75464	1136	.59	.29068	2092	.94	.95995	13390
.25	3.76614	1150	.60	.31214	2146	.95	6.11831	15836
.26	.77780	1166	.61	.33412	2198	.96	.31213	19382
.27	.78962	1182	.62	.35668	2256	.97	.56200	24987
.28	.80160	1198	.63	.37984	2316	.98	.91419	35219
.29	.81376	1216	.64	.40364	2380	.99	7.51625	60206
.30	.82608	1232	.65	.42812	2448	90°.00	Inf.	
.31	.83856	1248	.66	.45330	2518			
.32	.85124	1268	.67	.47922	2592			
.33	.86412	1288	.68	.50594	2672			
.34	.87718	1306	.69	.53352	2758			

PRICE 50 CENTS